Meaningful Forms and Fuzzy Geometry I: MPB & MGP

Mehdi Asasian$^{1,2*}$

$^1$Department of Applied Mathematics, Sharif University of Technology, Tehran, Iran. $^2$Department of Painting, University of Culture and Art, Sari, Iran.

Author’s contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Abstract

The analysis and critique of the artworks have always been of interest to artists. In this paper, we study the analysis and critique of artworks from the perspective of fuzzy logic. We try to provide definitions for Meaningful point and line in visual arts with the help of fuzzy geometry. We define Meaningful golden point (MGP) using the Meaningful painting box (MPB). With providing fuzzy definitions for visual elements, our modeling strives to obtain geometric forms that carry the artist’s mental-behavioral concepts. As a result, the way is paved for purposes such as analysis and critique of the artistic forms of modern and contemporary artworks, educational systems, art economics, and fuzzy geometric aesthetic. According to the features of fuzzy logic, our definitions and methods cover the deterministic mode. It is fully efficient in the analysis and critique of classical artworks.

Keywords: Fuzzy logic; fuzzy geometry; formalism; Kant; Tatarkiewicz.

1 Introduction

From the late nineteenth century with the development of modernist approaches to art, an unprecedented transformation was created in the approach to art. Theorists such as Clive Bell and Roger Fry considered form and formalism as the fundamental features of contemporary art [1,2]. They believed that the value of
artistic works is related to their formal qualities, regardless of the socio-historical conditions of art. In this way, this theory stood against the theory of representation. It has led to the emergence of artists such as Jackson Pollock and Willem de Kooning by abstracting the artworks, especially in the fields of painting and sculpturing. Throughout the history of art, the relationship between form and content has always been of interest to the artists. But it found a systematic form and helped shaping a kind of formalism, especially in the fields of beauty and art with Kant's ideas.

Using the ideas of Baumgarten, Kant, Schelling and Schiller; Tatarkiewicz has studied the genealogy of the form and has introduced various definitions of the form and its relevance to the content in his research. Clive Bell also posed a Significant form theory in the twentieth century [3]. Roger Fry emphasized the importance of the form. A new division was presented in the field of the explicit and hidden formalisms by Richard Wolheim [4]. On the other hand, Clement Greenberg tried to reorganize the art critique in the light of a form and revising Kant's third critique on this subject, stating that the origins of artistic formalism should be sought in the thoughts of Kant [5]. In the history of aesthetics, according to Tatarkiewicz, there are five meanings for a form [6], that the fifth meaning of the it was used by Kant. According to Kant and his followers, form is the share of mind in the subject and perception. In this sense, form is what the mind shapes by observation and experience of a phenomenon. So form is one of the characteristics of the mind, so it allows us to get things and phenomena in a specific form. This Kantian form is a priori concept. In fact, our mind imposes the form on phenomena. In the book of *Critique of Pure Reason*, he discovers the prior forms of science within our minds. Forms such as time and space and categories such as essence and causality are in this category [7].

In the meantime, in the book *Art & Illusion* [8], in the study of psychology of visual representation, Ernst Hans Gombrich investigated the roots of ambiguity and illusion in the artworks created in the history of art which is important in analyzing the form of artworks. With the formalist perspective emerged in the twentieth century, it is necessary to address the issue of "illusion" and the roots of ambiguity resulting from the efforts of the mind to receive and represent a phenomenon along with factors such as the meaning and content of the artworks. Eventually, these factors impose content on the visual works that affects the form. Ignoring this important factor, poses a problems to form critique. Because in analyze the artwork, we always expect those implied factors in the form to be decrypted by the audience. But the important point is that using the current Aristotelian geometric instrument that analyzes classical artworks ignoring the factors influencing the form and the modern and postmodern artworks cannot be analyzed properly. So we need a logic and geometry that put meaning and content on the form without harming the Kantian individuality contained in visual works. It should also provide tools and standards for the geometric aesthetic in art.

Here, based on the fuzzy logic proposed by Professor Zade [9], we attempted to deal with the visual elements forming visual forms in a different way. In the field of fuzzy thinking, researchers such as Bart Kosko (Kosko, 1993) investigated in their studies on the fuzzy logic and deals with different human thought categories, such as aesthetic foundations. They believe that fuzzy thinking is much more consistent with the human thinking compared to binary thinking. We believe that fuzzy thinking can cover a wide range of mental, visual and verbal ambiguities and processes nature. In this way, using the fuzzy geometry derived from this new logic, we try to define Meaningful geometric forms. We first define the Meaningful point and line, and finally develop the Meaningful golden points and painting box with the help of Meaningful golden ratio definition. So we can put the content on the geometric form using a mathematical model and try to obtain tools to critique and create a modern and postmodern artwork by developing the common geometric tools in the formal critique of the classical artworks. It is obvious (given the fact that the definitions and the theorems derived from fuzzy logic also cover two deterministic and stochastic modes) our definitions of Meaningful visual elements, in deterministic form, become classical definitions. Therefore it covers not only modern works but also classical artworks.

It is clear that discovery of the logical and mathematical mentally-behavioral models of the artists have advantages including the production of aesthetic criteria, better critique of artworks, contextualization for a better use of artwork by the economics and helping the art economics, the production of software to process and analyze the artworks, and finally helping to improve the methods of teaching modern and contemporary
arts. In the following, we will present our definitions of visual elements. Note that, for entering the arena of critique, we needed a fundamental and developed definition of the visual basic elements, definitions that were especially applicable, not merely theoretical, and understandable to artists as far as possible. The main purpose of the series of papers is to provide basic definitions and create the intended new basic links in the mind of the audience, and the examples provided are to better understand the concepts involved and I have postponed the aspect of critique until the full presentation of other fundamental concepts.

2 Meaningful Visual Point

In the basics of visual art, the point (unlike its Aristotelian mathematics definition) is defined as the simplest and most unreducible unit in visual communication. As can be seen, the mathematical definition of the point cannot express the semantic load and mental-behavioral ambiguities of the artist in dealing with the point. On the other hand, the distance from the mathematical definition of the point causes problems to the evaluation and analysis of modern and contemporary artworks. In order to preserve the individuality present in the modern art to express a Meaningful point, fuzzy logic is used instead of Aristotelian logic, and according to the works done by Eslami and Buckley [10,11], Debdas Ghosh, Debjani Chakraborty [12,13,14] in fuzzy geometry, we present our definitions for Meaningful visual point.

Definition 2.1 A Meaningful point (MP) at \((a, b)\) in \(\mathbb{R}^2\) (or painting box), written \(\tilde{P}(a,b)\), is defined its Meaningful function (MF):

1. \(\mu((x, y) \mid \tilde{P}(a,b))\) is upper semi-continuous;
2. \(\mu((x, y) \mid \tilde{P}(a,b)) = 1\) if and only if \((x, y) = (a, b)\);
3. \(\tilde{P}(\alpha)\) is a compact, convex subset of \(\mathbb{R}^2\) for all \(0 \leq \alpha \leq 1\).

Here, it should be noted that the following definition can also be made for a Meaningful visual point. Although it does not give us an imagination of the Meaningful function but it can be useful in future works to critique some of the artworks.

Definition 2.2. A Meaningful point (MP) is a pair \((\tilde{X}, \tilde{Y})\) Where \(\tilde{X}\) and \(\tilde{Y}\) are real fuzzy numbers.

In visual arts, our definition has the following advantages in comparison with the classical definition of point:

1. The mathematical properties of the Meaningful function can be used as aesthetic criteria for an artwork, and assess its degree of beauty and harmony.
2. Using this definition, we can create a path for the formal-semantic teaching of modern and post-modern artworks.
3. Extracting the Meaningful functions for each artists, geometric criteria used to critique and compare the works of an artist with himself and others. It can be provided according to harmony and the form of Meaningful functions used in their artworks (depending on how much their functions have the features of Meaningful functions of the above definition).
4. This definition is effective for the evaluation of stochastic-based artworks, such as Marcel Duchamp’s artworks. The Meaningful function in such stochastic works includes the artist's thoughts, and the stochastic process of creating the artworks.
5. Due to the properties of fuzzy logic, the definition can cover all verbal ambiguities in form and color.
6. Considering that at the time of artwork creation, the Meaningful functions are defined by the artist himself (according to the conditions of artwork creation and his purposes). So this definition protects the feature of Kantian individuality in modern and postmodern art. It is so important for us in future.

In this way, it can be said that the Meaningful point is the area of box to which a meaning function is assigned. It can be the bearer of ideas, visual ambiguities, and the mental-behavioral content of the artist. We
discuss the extraction of the Meaningful functions, and give examples in the modeling discussion in future works. Before turning to the definition of a Meaningful golden point, we need to provide a definition of a Meaningful line.

3 Meaningful Visual Line

On the basics of visual arts, the line is defined based on the point. Line is moving point or the history of the movement of a point. It is seen that the problem in defining the visual point has found, way to the definition of the visual line. Here we provide a definition and schematic example for the Meaningful visual line.

**Definition 3.1.** Let $\vec{P}_1$ and $\vec{P}_2$ be two Meaningful points in the plane. Define (for $0 \leq \alpha \leq 1$)

$$
\Omega (\alpha) = \left\{ (x, y) : \frac{x - u_1}{u_2 - u_1} = \frac{y - v_1}{v_2 - v_1}, (u_1, v_1) \in \vec{P}_1(\alpha), (u_2, v_2) \in \vec{P}_2(\alpha) \right\}
$$

Meaningful line (ML): $\vec{L}$ is

$$
\mu((x, y) | \vec{L}(a, b)) = \sup \{ \alpha : (x, y) \in \Omega (\alpha) \}
$$

**Example 3.1.** A ML may have figure like Fig. 1, where it passing through three Meaningful points.

![Image 1](image1)

**Fig. 1.** A meaningful line passing through three meaningful points

We will also use the Fig. 2 in analyzing artworks:

![Image 2](image2)

**Fig. 2.** The area that describes the meaningful line
4 Meaningful Golden Ratio

The composition of most classical works is based on the golden ratio rule. The golden ratio is the deterministic number of 1.618 which is widely used in classical artworks to produce beautiful ratios. But in modern artworks, these divisions and proportions became more and more individualized, and this forced us to define a Meaningful golden ratio.

Definition 4.1. The Meaningful golden ratio is the same as the fuzzy number of \( \sim 1.618 \), such that its membership function is artist's MF.

This definition helps the artist to incorporate his mental-behavioral and verbal meanings of the golden ratio into the Meaningful function, thereby helping to critique modern and postmodern artworks. Obviously, considering the characteristics of fuzzy logic, the Meaningful golden ratio in the deterministic mode also covers classical definition of golden ratio. In the following, we present two box divisional methods, one based on the Meaningful golden ratio and the other based on the Meaningful point. But first, we need to provide a definition for the Meaningful painting box.

5 Meaningful Painting Box

Definition 5.1. A Meaningful painting box (MPB) is a box with fuzzy dimensions or made up with Meaningful lines. In other words, MPB is a box with Meaningful functions for its edges.

Example 5.1. Suppose a Meaningful rectangular box with fuzzy dimensions of \( \tilde{a} \) and \( \tilde{b} \) units. We can imagine Fig. 3 for this box. Such that, the central lines (in each edges) make a classical box with deterministic dimensions of \( a \) and \( b \) units.

![Fig. 3. Meaningful painting box with meaningful Gaussian symmetric functions](image)

Note 5.1. The above figure is created based on the alpha-cut of the Meaningful functions corresponding edges. It is just a suggestion for a schematic dealing with MPB. Its shape is very dependent on artist's idea and alpha-cut of his MF. This flexible definition is useful in analyzing and criticizing artworks in future.

Note 5.2. The Meaningful functions of all four edges are not needed to be the same. This definition giving us a very flexible box. For example, we can define the asymmetric functions for box as follows:
Fig. 4. Box with meaningful Gaussian and triangular asymmetric functions

6 Meaningful Golden Point

In a rectangular MPB, by using of the fuzzy division of length and width by a Meaningful golden ratio 1.618, we can draw horizontal and vertical Meaningful guiding lines and obtain Meaningful golden point (MGP) such as Fig. 5.

Fig. 5. Horizontal and vertical meaningful guiding lines and meaningful golden point

One of the advantages of Meaningful box definition is the study of the works created in the field of post-modern art. Especially, the fields of Installation art and New Media. For example, if we have a rectangular box with a neon lamp, we do not get it as a deterministic rectangle in visual and perceptual terms. In a post-perceptual process, we decide on a rectangular box that includes the artist's mental meaning of the box, which can be modeled with Meaningful box.

Example 6.1. In this example, we will come up with another approach to dealing with a MGP. Consider a rectangular box with deterministic dimensions of a and b cm. Divide the length and width by three and pass the Meaningful guiding lines through the resulted points using your ML definition and the membership function of number three. It is seen that from the confluence of Meaningful guiding lines, we obtain MGP such as Fig. 6.
7 Modeling and Analyze

Construction of MF to create an artwork or its critique, is entirely dependent on the process of creation of the artwork such as the ambiguities or stochastic events. We will discuss the methods of constructing the Meaningful functions and present details in future works. Now we present our modeling with following examples. Note that the examples provided are to better understand the concepts involved and we have postponed the aspect of critique until the full presentation of other fundamental concepts, in future.

**Example 7.1.** Consider artwork by VIGUD which is presented in Fig. 7.

According to the above, we can obtain the Meaningful guiding lines and the MGP as shown in Fig. 8. The red lines are classical divisions of the painting box.
In the future works, we will propose methods for constructing MF based on artwork and define other visual elements to produce the critique of modern and postmodern artworks. Also, we will discuss the production of fuzzy aesthetic criteria based on Meaningful painting box. But now, for better clarification, consider Fig. 9. The painting is the result of the author's mind semi-abstract collision of the nature. In a project called "Summer", we tried to provide a form of meaningful line in deterministic box as practical examples.

A linear form of painting 7.3 is presented in Fig. 10. In this, the meaningful guiding line has crossed approximately $1/3$ (or $\frac{1}{3}$) of the width of the box.

In Fig. 11, we have presented a piece of the Meaningful asymmetric Gaussian function that describes the Fig 10.
8 Conclusion

The analysis and critique of the artworks have always been of interest to artists. Hence, the tools and methods that can effectively examine modern and postmodern artworks are so important. In this paper, using the fuzzy logic and fuzzy geometry, we provided a new definition for the visual elements, point and line. We have achieved a mathematical model for geometric forms that can carry concept and stochastic-based events in painting. We have provided basic tools such as meaningful form and meaningful golden point to help in the creation and analysis of modern and contemporary artworks base on the Kantian individuality. It is very important for us that the presented definitions can cover the classical artworks and be consistent with its formal critique. Obviously, finding mental-behavioral models of artists can also help in educational goals and better economic exploitations.

Competing Interests

Author has declared that no competing interests exist.

References


