Intuitionistic Fuzzy Nano Generalized Closed Sets

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Authors’ contributions

This work was carried out in collaboration between both authors. Author ASAR designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author MR managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper we introduce and study the notion of intuitionistic fuzzy nano generalized closed sets in intuitionistic fuzzy nano topological spaces and some of its properties and results. A real time application of intuitionistic fuzzy nano topology under intuitionistic fuzzy nano upper approximation in multi criterion decision making is discussed with an example.

Keywords: Intuitionistic fuzzy nano closed sets; intuitionistic fuzzy nano generalized closed sets; multi criterion decision making.

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1 Introduction


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2 Preliminaries

Definition 2.1 [7]: Let \( U \) be the universe, \( R \) be an equivalence relation on \( U \) and \( \tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\} \) where \( X \subseteq U \). Then by property 1.2, \( \tau_R(X) \) satisfies the following axioms:

1. \( U \) and \( \emptyset \in \tau_R(X) \).
2. The union of the elements of any subcollection of \( \tau_R(X) \) is in \( \tau_R(X) \).
3. The intersection of the elements of any finite subcollection of \( \tau_R(X) \) is in \( \tau_R(X) \).

That is, \( \tau_R(X) \) is a topology on \( U \) called the nano topology on \( U \) with respect to \( X \). We call \( (U, \tau_R(X)) \) as the nano topological space. The elements of \( \tau_R(X) \) are called as nano-open sets. If \( (U, \tau_R(X)) \) is a nano topological space \( [8] \) where \( X \subseteq U \) and \( \text{if} \ A \subseteq U \), then the nano interior of \( A \) is defined as the union of all nano-open subsets of \( A \) and it is denoted by \( N\text{Int}(A) \). \( N\text{Int}(A) \) is the largest nano-open subset of \( A \). The nano closure of \( A \) is defined as the intersection of all nano closed sets containing \( A \) and it is denoted by \( NC(I)(A) \). That is, \( NC(I)(A) \) is the smallest nano closed set containing \( A \).

Definition 2.2 [7]: Let \( (U, \tau_R(X)) \) and \( (V, \tau'_R(Y)) \) be two nano topological spaces. Then a mapping \( f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y)) \) is nano continuous on \( U \) if the inverse image of every nano-open set in \( V \) is nano-open in \( U \).

Definition 2.3 [9]: Let \( (U, \tau_R(X)) \) be a nano topological space. A subset \( A \) of \( (U, \tau_R(X)) \) is called nano generalized closed set (briefly Ng- closed) if \( NC(I)(A) \subseteq V \) where \( A \subseteq V \) and \( V \) is nano open.

Definition 2.4 [10]: Let \( U \) be a non-empty finite set of objects called the universe and \( R \) be an intuitionistic fuzzy equivalence relation on \( U \) named as the indiscernibility relation. Then \( U \) is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair \( (U, R) \) is said to be the intuitionistic fuzzy approximation space (in short IFAS). Let \( X \subseteq U \).

1. The lower approximation of \( X \) with respect to \( R \) is the set of all \( x \) which can be for certain classified as \( x \) with respect to \( R \) and it is denoted by \( IFL_R(X) \). That is, \( IFL_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\} \), where \( R(x) \) denotes the equivalence class determined by \( x \in U \).
2. The upper approximation of \( X \) with respect to \( R \) is the set of all \( x \) which can be possibly classified as \( x \) with respect to \( R \) and it is denoted by \( IFU_R(X) \). That is, \( IFU_R(X) = \bigcup_{x \in U} \{R(x) \cap X \neq \emptyset\} \).
3. The boundary region of \( X \) with respect to \( R \) is the set of all \( x \) which can be classified neither as \( X \) nor as not-\( X \) with respect to \( R \) and it is denoted by \( IFB_R(X) \). That is, \( IFB_R(X) = IFU_R(X) - IFL_R(X) \).

Let \( U \) be the universe, \( R \) be an intuitionistic fuzzy equivalence relation on \( U \) and \( \tau_R(X) = \{1, 0 \sim, IFL_R(X), IFU_R(X), IFB_R(X)\} \) where \( X \subseteq U \). Then \( \tau_R(X) \) satisfies the following axioms:
(1) $1 \sim 0$ in $\tau_R(X)$.
(2) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on $U$ called the intuitionistic fuzzy nano topology on $U$ with respect to $X$. We call $(U, \tau_R(X))$ as the intuitionistic fuzzy nano topological space. The elements of $\tau_R(X)$ are called as intuitionistic fuzzy nano-open sets. If $(U, \tau_R(X))$ is an intuitionistic fuzzy nano topological space (In short IFNTS) where $X \subseteq U$ and if $A \subseteq U$, then the intuitionistic fuzzy nano interior of $A$ is defined as the union of all intuitionistic fuzzy nano-open subsets of $A$ and it is denoted by $\text{IFNInt}(A)$. $\text{IFNInt}(A)$ is the largest intuitionistic fuzzy nano-open subset of $A$. The intuitionistic fuzzy nano closure of $A$ is defined as the intersection of all intuitionistic fuzzy nano closed sets (In short IFNCS) containing $A$ and it is denoted by $\text{IFNCI}(A)$. That is, $\text{IFNCI}(A)$ is the smallest intuitionistic fuzzy nano closed set containing $A$.

3 Intuitionistic Fuzzy Nano Generalized Closed Set

Throughout this paper $(U, \tau_R(X))$ is an intuitionistic fuzzy nano topological space with respect to $X$ where $X \subseteq U$, $R$ is an equivalence relation on $U$, $U/R$ denotes the family of equivalence classes of $U$ by $R$.

Definition 3.1: Let $(U, \tau_R(X))$ be an intuitionistic fuzzy nano topological space. A subset $A$ of $(U, \tau_R(X))$ is called intuitionistic fuzzy nano generalized closed set (briefly IFNg-closed) if $\text{IFNCI}(A) \subseteq V$ where $A \subseteq V$ and $V$ is intuitionistic fuzzy nano open (In short IFNOS).

Example 3.2: Let $(U, R)$ be an intuitionistic fuzzy approximation space (In short IFAS) where $U = \{a_1, a_2, a_3\}$ with $R = \{(a_1, a_1), (a_1, a_2), (a_1, a_3), (a_2, a_1), (a_2, a_2), (a_2, a_3), (a_3, a_1), (a_3, a_2), (a_3, a_3)\}$.

Let $A = \{(a_1, 0.7, 0.3), (a_2, 0.5, 0.5), (a_3, 0.6, 0.3)\}$ be an intuitionistic fuzzy set (In short IFS) on $U$ then $\tau_R(X) = \{1, 0\}$ in $\text{IFNCI}(A)$.

Let $B = \{(a_2, 0.5, 0.5), (a_2, 0.6, 0.3)\}$ be an IFS such that $B \subseteq V$ then $\text{IFNCI}(B) = \{(a_2, 0.5, 0.5), (a_2, 0.6, 0.3)\} \subseteq V$. That is $B$ is said to be IFNg-closed in $(U, \tau_R(X))$.

Theorem 3.3: A subset $B$ of $(U, \tau_R(X))$ is IFNg-closed if $\text{IFNCI}(B) \subseteq B$ contains no empty IFS.

Proof: Suppose if $B$ is IFNg-closed. Then $\text{IFNCI}(B) \subseteq V$ where $B \subseteq V$ and $V$ is IFNO. Let $Y$ be a intuitionistic fuzzy nano closed subset of $\text{IFNCI}(B) = B$. Then $B \subseteq Y^c$ and $Y$ is IFNO. Since $B$ is Ng-closed, $\text{IFNCI}(B) \subseteq Y^c$ or $Y \subseteq (\text{IFNCI}(B))^c$. That is, $Y \subseteq \text{IFNCI}(B)$ and $Y \subseteq (\text{IFNCI}(B))^c$ implies that $Y \subseteq \emptyset$. So $Y$ is empty.

Theorem 3.4: If $A$ and $B$ are IFNg-closed, then $A \cup B$ is IFNg-closed.

Proof: Let $A$ and $B$ are IFNg-closed sets. Then $\text{IFNCI}(A) \subseteq V$ where $A \subseteq V$ and $V$ is IFNO and $\text{IFNCI}(B) \subseteq V$ where $B \subseteq V$ and $V$ is IFOS. Since $A$ and $B$ are subsets of $V$, $A \cup B$ is a subset of $V$ and $V$ is IFNS. Then $\text{IFNCI}(A \cup B) = \text{IFNCI}(A) \cup \text{IFNCI}(B) \subseteq V$ which implies that $A \cup B$ is IFNg-closed.

Theorem 3.5: If $C$ is IFNg-closed and $C \subseteq D \subseteq \text{IFNCI}(C)$, then $D$ is IFNg-closed.

Proof: Let $D \subseteq V$ where $V$ is IFNS in $\tau_R(X)$. Then $C \subseteq D$ implies $C \subseteq V$. Since $C$ in IFNg-closed, $\text{IFNCI}(A) \subseteq V$. Also $C \subseteq \text{NCl}(D)$ implies $\text{IFNCI}(D) \subseteq \text{IFNCI}(C)$. Thus $\text{IFNCI}(D) \subseteq V$ and so $D$ is IFNg-closed.
Theorem 3.6: Every IFNCS is a IFNg-closed set.

Proof: Let B ⊂ V and V is IFNOS in τ_φ(X). Since B is IFNCS, IFNCl(B) ⊂ B. That is IFNCl(B) ⊂ B ⊂ V. Hence B is a IFNg-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.7: Let (U, R) be an IFAS where U = {u, v, w} with

\[ R = \{(u, 0.5, 0.3), (v, 0.5, 0.4), (w, 0.4, 0.3)\} \]

Let B = {(w, 0.5, 0.3), (v, 0.5, 0.4), (w, 0.4, 0.3)} be an intuitionistic fuzzy set (In short IFS) on U then τ_φ(X) = {1, 0, 1, 0, 1}. Let V = {(u, 0.5, 0.3), (v, 0.5, 0.3), (w, 0.4, 0.3)} be an IFS in U and

IFNCl(B) = \{(a_1, 0.5, 0.5), (a_2, 0.5, 0.5), (a_3, 0.3, 0.6)\} ⊂ V. That is B is said to be IFNg-closed in (U, τ_φ(X)). But B is not IFNCS.

Theorem 3.8: An IFNg-closed set A is IFNCS if and only if IFNCl(A) – A is IFNCS.

Proof: (Necessity) Let A is IFNCS. Then IFNCl(A) = A and so IFNCl(A) – A = ø which is IFNCS.

(Sufficiency) Suppose IFNCl(A) – A is IFNCS. Then IFNCl(A) – A = ø since A is IFNCS. That is, IFNCl(A) = A or A is IFNCS.

Theorem 3.9: Suppose that B ⊂ A ⊂ U, B is an IFNg-closed set relative to A and that A is an IFNg-closed subset of U. Then B is IFNg-closed relative to U.

Proof: Let B ⊂ V and suppose that V is IFNOS in U. Then B ⊂ A ∩ V. Therefore IFNCl(B) ⊂ A ∩ V. It follows that A ∩ IFNCl(B) ⊂ A ∩ V and A ⊂ V ∩ IFNCl(B). Since A is IFNg-closed in U, we have IFNCl(A) ⊂ V ∩ IFNCl(B). Therefore IFNCl(B) ⊂ IFNCl(A) ⊂ V ∩ IFNCl(B) and so IFNCl(B) ⊂ V. Then B is IFNg-closed relative to V.

Corollary 3.10: Let A be a IFNg-closed set and suppose that F is a IFNCS. Then A ∩ F is a IFNg-closed set.

Theorem 3.11: For each a ∈ U, either \{a\} is IFNCS (or) \{a\}^C is IFNg-closed in τ_φ(X).

Proof: Suppose \{a\} is not IFNCS in U. Then \{a\}^C is not IFNOS and the only IFNOS containing \{a\}^C is V ⊂ U. That is \{a\}^C ⊂ U. Therefore IFNCl(\{a\}^C) ⊂ U which implies \{a\}^C is IFNg-closed in τ_φ(X).

Theorem 3.12: Let A be a g-closed subset of (X, τ). Then Cl(A) – A does not contain any non-empty closed sets.

Proof: Let F ∈ C(X) such that F ⊂ Cl(A) – A. Since X – F is open, A ⊂ X – F and A is g-closed, it follows that Cl(A) ⊂ X – F and thus F ⊂ X – Cl(A). This implies that F ⊂ (X – Cl(\{\})) ∩ (Cl(A) – A) = ø and hence F = ø.
Corollary 3.13: Let A be IFNg-closed set. Then A is IFNC if and only if \(IFNC(A) = A\) is IFNCS.

**Proof:** Let A be IFNg-closed set. If A is IFNCS, then we have \(IFNC(A) = A = \emptyset\) which is IFNCS. Conversely, let \(IFNC(A) = A\) be IFNCS. Then, by Theorem 3.12, \(IFNC(A) = \emptyset\) does not contain any non-empty IFNC subset and since \(IFNC(A) = A\) is IFNC subset of itself, then \(IFNC(A) = A = \emptyset\). This implies that \(A = IFNC(A)\) and so A is IFNCS.

4 A Real Time Application

In this section, we discuss a real time application of IFNTS on one or more universal sets to multi criterion decision making. The application is discussed using intuitionistic fuzzy nano upper approximation space. In the case of telecommunication industries, there are various factors like tariff plans, affordable premium, network coverage, quality of service and location of service centres, which influence the public people interest on the company. Hence, IF relation provides the better relation between the public and telecommunication industries.

Consider \(V = \{a_1, a_2, a_3, a_4, a_5\}\), in which \(a_1\) is tariff plans; \(a_2\) is affordable premium; \(a_3\) is network coverage; \(a_4\) is quality of service; \(a_5\) is location of service centres and decisions \(U = \{b_1, b_2, b_3, b_4, b_5\}\), in which \(b_1\) is excellent; \(b_2\) is best; \(b_3\) is good; \(b_4\) is satisfactory; \(b_5\) is least affordable. Investors from various financial status are invited to the survey. Therefore, \((U, V, IFU, IFL)\) be an IFAS, where 
\[U = \{b_1, b_2, b_3, b_4, b_5\}\] and 
\[V = \{a_1, a_2, a_3, a_4, a_5\}\].

If 17% investors rate excellent and 9% rate not excellent; 30% rate best; 3% rate not best; 15% give good; 10% rate not good; 10% rate satisfactory; 18% give not satisfactory; 24% give least affordable and 20% give not affordable, then we have the vector can be obtained as 
\[(.17,.09,.3,.2,.6,.1,.1,.5,.23,.1)\], \[(.3,.03,.37,.25,.35,.3,.3,.2,.2,.1)\], 
\[(.15,.1,.35,.15,.2,.3,.15,.4,.3,.4)\], 
\[(.1,.1,.2,.25,.13,.3,.4,.36,.2,.1,.3)\], 
\[(.24,.2,.3,.5,.1,.2,.4,.15,.2,.4)\].

Based on the decision vectors, the IF relation from U to V is given by the following matrix.

\[
\begin{bmatrix}
-17 & .09 & .3 & .2 & .6 & .1 & .1 & .5 & .23 & .1 \\
.3 & .03 & .37 & .25 & .35 & .3 & .3 & .2 & .2 & .1 \\
.15 & .1 & .35 & .15 & .2 & .3 & .15 & .4 & .3 & .4 \\
.1 & .1 & .25 & .13 & .3 & .4 & .36 & .2 & .1 & .3 \\
.24 & .2 & .3 & .5 & .1 & .2 & .4 & .15 & .2 & .4 
\end{bmatrix}
\]

Two category of investors are considered, where right weightage for each criterion in V are \(V_1 = (b_{1,1},.2), b_{2,6},.13, b_{3,20},.3, b_{4,15},.05, b_{5,1},.25)\) and \(V_2 = (b_{1,1},.3, b_{2,5},.15, b_{3,20},.4, b_{4,13},.32, b_{5,12},.31)\) respectively. Thus, by using IF upper approximation we have:

\[
IF_{U}(V_1) = (b_{1,1},.3,0.2, b_{2,30},.7, b_{3,15},.05, b_{4,25},.013, b_{5,0.2},.20)\] and 
\[
IF_{U}(V_2) = (b_{1,1},.4,0.2, b_{2,35},.25, b_{3,2},.014, b_{4,3},.015, b_{5,24},.03)\]

respectively.
From above, according to the principle of maximum membership, the decision for the first category of investors is best whereas for the second category is excellent.

5 Conclusion

We introduced intuitionistic fuzzy nano generalized closed sets in intuitionistic fuzzy nano topological spaces and an application of intuitionistic fuzzy nano topology in Multi Criterion Decision Making.

Competing Interests

Authors have declared that no competing interests exist.

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