Radiative Effects on Unsteady MHD Couette Flow through a Parallel Plate with Constant Pressure Gradient

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Authors¹ contributions

This work was carried out in collaboration among all authors. Author ROO designed the study. Author EOA performed the model analysis, wrote the protocol and wrote the first draft of the manuscript. Authors MDS and AL managed the analyses of the study. Author EOA managed the literature searches. All authors read and approved the final manuscript.

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Abstract

In this paper, the unsteady MHD Couette flow through a porous medium of a viscous incompressible fluid bounded by two parallel porous plates under the influence of thermal radiation and chemical reaction is investigated. A uniform suction and injection are applied perpendicular to the plates while the fluid motion is subjected to the constant pressure gradient. The transformed conservation equations are solved analytically subject to physically appropriate boundary conditions by using the Eigenfunction expansion technique. The influence of some emerging non-dimensional parameters namely, pressure gradient, suction parameter, radiation parameter, and Hartman number are examined in detail. It is observed that the primary velocity is increased with increasing pressure gradient while the increase in radiation parameter leads to a decrease in the thermal profile of the flow.

Keywords: Eigenfunction expansion technique; magnetohydrodynamics (MHD); constant pressure gradient; suction; hall current.

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1 Introduction

The dynamics of fluids through the porous channel have been a popular area of research regarding to numerous increasing applications in chemical, mechanical, and material process engineering. Examples of such fluid include clay coating, coal, oil slurries, shampoo, paints cosmetic products, grease, custard and physiological liquids (blood, bile, and synovial fluid). Over the years, considerable interest has been observed on the effect of MHD in viscous, incompressible, non-Newtonian fluid flow with heat transfer. These interests on non-Newtonian fluids are owed to its important applications in various branches of science, engineering and technology, particularly in chemical and nuclear industries, material processing, geophysics and bio-engineering. In view of these applications, an extensive range of mathematical models have been developed to simulate the diverse hydrodynamic behavior of these non-Newtonian fluids. However, different non-Newtonian fluid models have been presented by researchers and solved using various types of analytical and computational schemes.


The aim of the research is to establish an analytical solution capable of describing the concentration, temperature and velocity in the process of MHD Couette flow through a parallel plate with constant pressure gradient.

2 Mathematical Formulation

Following Sayed-Ahmed et al. [1], the unsteady flow of a viscous, incompressible, non-conducting fluid through a channel with chemical reaction, thermal radiation, constant and variable pressure gradient in the presence of magnetic field is investigated. The flow is assumed to be laminar, incompressible and flows between two infinite horizontal plates located at \( y = \pm h \) which extends from \( x = -\infty \) to \( \infty \) and from \( z = -\infty \) to \( \infty \).
The upper plate is suddenly set into motion and moves with a uniform velocity $U_0$ while the lower plate is kept stationary as shown in the diagram below. The upper plate is simultaneously subjected to a step change in temperature from $T_1$ to $T_2$. The upper and lower plates are kept at two constant temperatures $T_2$ and $T_1$ respectively with $T_2 > T_1$. The fluid flows between the two plates under the influence of an exponential decaying with time pressure gradient in the x-direction which is a generalization of a constant pressure gradient. A uniform suction from above and injection from below with constant velocity $V_0$ which are all applied at $t = 0$. The system is subjected to a uniform magnetic field $B_0$ in the positive y-direction and is assumed undisturbed as the induced magnetic field is neglected by assuming a small magnetic Reynolds number. The Hall effect is taken into consideration and consequently a z-component of the velocity is expected to arise.

![Fig. 1. Schematic diagram of the problem](image)

Based on the above assumptions,

$$v = ui + v_0j + wk$$

(1)

Introducing a Chapman-Rubesin viscosity law, with $w = 1$ as shown in Olayiwola (2016) and using the condition at the lower plate, results in:

$$\mu = \frac{c\mu_iT}{T_1}$$

(2)

Where $\mu_i$ is the Casson coefficient of viscosity.

Thus, the two components of the governing momentum equation in the dimensional form are as follows:

$$\rho \frac{\partial u}{\partial t} + \rho v_0 \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2}{(1+BiBe)^2 + Be^2} \left[ \left( 1 + BiBe \right) u + Be \right] -$$

$$\mu \frac{u}{k} + g \beta_T \left( T - T_1 \right) + g \beta_C \left( C - C_1 \right)$$

(3)
The energy equation in dimensional form is given as:

\[
\rho \cfrac{\partial w}{\partial t} + \rho \nu_0 \cfrac{\partial w}{\partial y} = \cfrac{\partial}{\partial y} \left( \mu \cfrac{\partial w}{\partial y} \right) - \frac{\sigma B_0^2}{(1 + BiBe) + Be^2} \left[ (1 + BiBe)w - Beu \right] - \frac{\mu w}{k} \tag{4}
\]

The energy equation in dimensional form is given as:

\[
\rho C_p \cfrac{\partial T}{\partial t} + \rho C_p \nu_0 \cfrac{\partial T}{\partial y} = \cfrac{1}{Pr} \cfrac{\partial}{\partial y} \left( \mu \cfrac{\partial T}{\partial y} \right) + \mu \left[ \left( \cfrac{\partial u}{\partial y} \right)^2 + \left( \cfrac{\partial w}{\partial y} \right)^2 \right] + \frac{\sigma B_0^2}{(1 + BiBe)^2 + Be^2} \left[ u^2 + w^2 \right] - \frac{1}{\rho C_p} \cfrac{\partial q}{\partial y} \tag{5}
\]

The concentration equation in dimensional form is given as:

\[
\rho \cfrac{\partial C}{\partial t} + \rho \nu_0 \cfrac{\partial C}{\partial y} = \cfrac{1}{Sc} \cfrac{\partial}{\partial y} \left( \mu \cfrac{\partial C}{\partial y} \right) + D_t \cfrac{\partial^2 T}{\partial y^2} - D_2 (C_2 - C_1) \tag{6}
\]

Subject to the initial and boundary conditions:

\[
\begin{align*}
&u(y, 0) = 0, \quad u(-h, t) = 0, \quad u(h, t) = U_0 \\
&w(y, 0) = U_0 y(1 - y), \quad w(-h, t) = 0, \quad w(h, t) = 0 \\
&T(y, 0) = 0, \quad T(-h, t) = T_1, \quad T(h, t) = T_2 \\
&C(y, 0) = 0, \quad C(-h, t) = C_1, \quad C(h, t) = C_2
\end{align*} \tag{7}
\]

Where \( \rho \) and \( \mu \) are respectively the density and apparent viscosity of the fluid, \( \delta \) is electric conductivity, \( \beta \) is Hall factor, \( Bi \) is ion slip parameter, \( Be = \sigma \beta H_0 \) is Hall parameter, \( c \) and \( k \) are the specific heat capacity and thermal conductivity of the fluid respectively. Where \( u \) and \( w \) are components of velocities along and perpendicular to the plate in \( x \) and \( y \) directions respectively, \( \beta_x \) is the coefficient of volume expansion of the moving fluid, \( \beta_C \) is the coefficient of volumetric expansion with concentration, \( \nu \) is the kinematic viscosity, \( T \) is the temperature of the fluid, \( C \) is the concentration of the fluid, \( C_1 \) is the concentration at infinity, \( D_t \) the thermal diffusivity, \( D_2 \) the chemical reaction rate constant, \( C_p \) is the specific heat capacity at constant pressure. \( t \) is the time, \( g \) is the gravitational force, \( \mu \) is the magnetic permeability of the fluid, \( K \) is the porous media permeability coefficient, \( q \) is radiative heat flux, \( H_0 \) is the intensity of the magnetic field, \( B_0 = \mu \beta H_0 \) is electromagnetic induction, \( \tau_0 \) is yield stress, \( \alpha \) is coefficient of volume expansion due to temperature and \( \alpha \) is the mean radiation absorption coefficient.

To write the governing dimensional equations (3)-(6) with their corresponding boundary conditions (7) in non-dimensional form, we use the following dimensionless variables:
\( \bar{u} = \frac{u}{U_0}, \quad \bar{w} = \frac{w}{U_0}, \quad \bar{y} = \frac{y}{h}, \quad \bar{x} = \frac{x}{h}, \quad \bar{t} = \frac{tU_0}{h}, \quad \theta = \frac{T-T_i}{T_f-T_i} \)

\( \varphi = \frac{C_2-C_1}{C_2-C_1}, \quad \bar{P} = \frac{P}{\rho U_0^2}, \quad \bar{\mu} = \frac{c\mu T}{T_i}, \quad \sigma (\text{pressure gradient}) = -\frac{\partial \bar{P}}{\partial \bar{x}} \)

and we obtain

\[ \frac{\partial u}{\partial t} + S \frac{\partial u}{\partial y} = \sigma + \frac{c}{\text{Re} \frac{\partial}{\partial y}} \left( (a\theta+1) \frac{\partial u}{\partial y} \right) - \frac{H_a^2}{\text{Re}(1+BiBe)^2} \left[ (1+BiBe)u + Be \right] - \frac{P_c}{\text{Re}} ((a\theta+1)u) + Gr\theta + Gr\phi \]

\[ \frac{\partial w}{\partial t} + S \frac{\partial w}{\partial y} = \frac{c}{\text{Re} \frac{\partial}{\partial y}} \left( (a\theta+1) \frac{\partial w}{\partial y} \right) - \frac{H_a^2}{\text{Re}(1+BiBe)^2} \left[ (1+BiBe)w - Be \right] - \frac{P_c}{\text{Re}} ((a\theta+1)w) \]

\[ \frac{\partial \theta}{\partial t} + S \frac{\partial \theta}{\partial y} = \frac{c}{\text{RePr} \frac{\partial}{\partial y}} \left( (a\theta+1) \frac{\partial \theta}{\partial y} \right) + \frac{cEc}{\text{Re}} (a\theta+1) \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] + \frac{EcH_a^2}{\text{Re}(1+BiBe)^2 + Be^2} [u^2 + w^2] - Ra^2 \theta \]

\[ \frac{\partial \phi}{\partial t} + S \frac{\partial \phi}{\partial y} = \frac{c}{Sc \text{Re} \frac{\partial}{\partial y}} \left( (a\theta+1) \frac{\partial \phi}{\partial y} \right) + T_d \frac{\partial^2 \theta}{\partial y^2} - Kr \phi \]

Subject to the initial and boundary conditions

\[ u(y,0) = 0, \quad u(-1,t) = 0, \quad u(1,t) = 1 \]

\[ w(y,0) = y(1-y), \quad w(-1,t) = 0, \quad w(1,t) = 0 \]

\[ \theta(y,0) = 0, \quad \theta(-1,t) = 0, \quad \theta(1,t) = 1 \]

\[ \phi(y,0) = 0, \quad \phi(-1,t) = 0, \quad \phi(1,t) = 1 \]
Where

\[
\begin{align*}
\text{Re} &= \frac{\rho U_0 h}{\mu}, \quad S = \frac{v_0}{U_0}, \quad P = \frac{h^2 \mu_0}{k}, \quad Ha^2 = \frac{\delta B_0^2 h^2}{\mu_0}, \quad Gr_\theta = \frac{g \beta_\theta (T_2 - T_1) h}{\rho U_0^2}, \\
Gr_\phi &= \frac{g \beta_\phi (C_2 - C_1) h}{\rho U_0^2}, \quad \Pr = \frac{\mu c_p}{k}, \quad Ec = \frac{U_0^2}{c_p (T_2 - T_1)}, \quad Ra^2 = \frac{4 \alpha^2 h}{\rho^3 C_\rho U_0}, \\
Sc &= \frac{U_0 h}{D}, \quad T_D = \frac{D(T_2 - T_1)}{h(C_2 - C_1) U_0}, \quad a = \frac{T_2 - T_1}{T_1}, \quad Kr = \frac{D h}{\rho U_0},
\end{align*}
\]

(14)

3 Method of Solution

3.1 Transformation

Since the boundary conditions are from -1 to 1, we first transform the boundary conditions to 0 to 1 using the transformation:

\[
z = \frac{y + 1}{2}
\]

(15)

Let \(0 < S < 1\) and \(Ec = bS, \quad a = eS, \quad Be = fS, \quad Gr_\theta = gS, \quad Gr_\phi = hS\) such that

\[
\begin{align*}
u(z,t) &= u_0(z,t) + S u_1(z,t) + \ldots \\
w(z,t) &= w_0(z,t) + S w_1(z,t) + \ldots \\
\theta(z,t) &= \theta_0(z,t) + S \theta_1(z,t) + \ldots \\
\phi(z,t) &= \phi_0(z,t) + S \phi_1(z,t) + \ldots
\end{align*}
\]

(16)

Collecting like powers of \(S\), we have for:

\[
S^0: \quad \frac{\partial \theta_0}{\partial t} = \frac{c}{4 \text{Re} \Pr} \frac{\partial}{\partial z} \left[ \frac{\partial \theta_0}{\partial z} \right] - Ra^2 \theta_0
\]

(17)

\[
\begin{align*}
\theta_0(z,0) &= 0, \quad \theta_0(0,t) = 0, \quad \theta_0(1,t) = 1
\end{align*}
\]
\[
\frac{\partial \phi_0}{\partial t} = \frac{c}{4Sc \text{Re}} \frac{\partial}{\partial z} \left( \frac{\partial \phi_0}{\partial z} \right) + T \frac{\partial^2 \theta_0}{\partial z^2} - Kr \phi_0 \\
\phi_0(z,0) = 0, \quad \phi_0(0,t) = 0, \quad \phi_0(1,t) = 1
\]

(18)

\[
\frac{\partial w_0}{\partial t} = \frac{c}{4 \text{Re}} \frac{\partial}{\partial z} \left( \frac{\partial w_0}{\partial z} \right) - \frac{Ha^2}{\text{Re}} \left[ \frac{w_0}{w_0} - \frac{Pc}{\text{Re}} w_0 \right]
\]

(19)

\[
w_0(z,0) = (2z-1)(2-2z), \quad w_0(0,t) = 0, \quad w_0(1,t) = 0
\]

\[
\frac{\partial u_0}{\partial t} = \frac{c}{4 \text{Re}} \frac{\partial}{\partial z} \left( \frac{\partial u_0}{\partial z} \right) - \frac{Ha^2}{\text{Re}} [u_0] - \frac{Pc}{\text{Re}} (u_0) + \sigma
\]

(20)

\[
u_0(z,0) = 0, \quad u_0(0,t) = 0, \quad u_0(1,t) = 1
\]

\[
S^1:
\frac{\partial u_1}{\partial t} + \frac{1}{2} \frac{\partial u_1}{\partial z} = \frac{c}{4 \text{Re}} \frac{\partial}{\partial z} \left( e \theta_0 \frac{\partial u_1}{\partial z} + \frac{\partial u_1}{\partial z} \right) - \frac{Ha^2}{\text{Re}} [u_0 \text{Bif} + u_1 + f w_0] - \frac{Pc}{\text{Re}} (e \theta_0 u_0 + u_1) +
\]

(21)

\[
g \theta_0 + h \phi_0
\]

\[
u_1(z,0) = 0, \quad u_1(0,t) = 0, \quad u_1(1,t) = 0
\]

\[
\frac{\partial \theta_1}{\partial t} + \frac{1}{2} \frac{\partial \theta_1}{\partial z} = \frac{c}{4 \text{Re} \text{Pr}} \frac{\partial}{\partial y} \left[ e \theta_0 \frac{\partial \theta_0}{\partial z} + \frac{\partial \theta_1}{\partial z} \right] + \frac{bc}{4 \text{Re}} \left[ \left( \frac{\partial u_0}{\partial y} \right)^2 + \left( \frac{\partial w_0}{\partial y} \right)^2 \right] - \frac{bHa^2}{\text{Re}} \left[ (u_0)^2 + (w_0)^2 \right] - Ra^2 \theta_1
\]

(22)

\[
\theta_1(z,0) = 0, \quad \theta_1(0,t) = 0, \quad \theta_1(1,t) = 0
\]
$$\frac{\partial w_i}{\partial t} + \frac{1}{2} \frac{\partial w_0}{\partial z} = \frac{c}{4 \text{Re}} \left( \frac{\partial^2 w_0}{\partial z^2} + \frac{\partial w_i}{\partial z} \right) - \frac{H a^2}{\text{Re}} \left[ w_0 \cdot B i f + w_i - f u_0 \right] - \frac{P c}{\text{Re}} \left( e \theta w_0 + w_i \right)$$

$$w_i(z,0) = 0, \quad w_i(0,t) = 0, \quad w_i(1,t) = 0$$

$$\frac{\partial \phi_i}{\partial t} + \frac{1}{2} \frac{\partial \phi_0}{\partial z} = \frac{c}{4 \text{Sc} \text{Re}} \left( \frac{\partial^2 \phi_0}{\partial z^2} + \frac{\partial \phi_i}{\partial z} \right) + T_p \frac{\partial^2 \theta_i}{\partial z^2} - K r \phi_i$$

$$\phi_i(z,0) = 0, \quad \phi_i(0,t) = 0, \quad \phi_i(1,t) = 0$$

3.2 Eigenfunction expansion technique

Now, consider the problem (see Myint-U and Debnath, 1987)

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} + \alpha u + F(x,t)$$

$$u(x,0) = f(x), \quad u(0,t) = 0, \quad u(L,t) = 0$$

For the solution of problem (25), we assume a solution of the form

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t) \sin \frac{n\pi}{L} x$$

Where

$$u_n(t) = \int_0^t e^{-k \left( \frac{a - \left( \frac{n\pi}{L} \right)^2}{\tau} \right)} F_n(\tau) d\tau + b_n e^{-k \left( \frac{a - \left( \frac{n\pi}{L} \right)^2}{\tau} \right)}$$

$$F_n(t) = \frac{2}{L} \int_0^L F(x,t) \sin \frac{n\pi}{L} x dx$$

$$b_n(t) = \frac{2}{L} \int_0^L F(x) \sin \frac{n\pi}{L} x dx$$
Comparing equation (17) – (24) with the (25) we obtain the solutions to the velocity (primary and secondary), temperature, and concentration distributions as

\[ \theta_0(z, t) = z + \sum_{n=1}^{\infty} q_1 \left(1 - e^{-q_1 t}\right) \sin n\pi z \]  
(30)

\[ \phi_0(z, t) = z + v_2(z, t) \]  
(31)

\[ w_0(z, t) = \sum_{n=1}^{\infty} q_{10} e^{-q_1 t} \sin \pi n z \]  
(32)

\[ u_0(z, t) = z + \sum_{n=1}^{\infty} \frac{q_{12}}{q_{11}} \left(1 - e^{-q_1 t}\right) \sin \pi n z \]  
(33)

\[ u_1(z, t) = \sum_{n=1}^{\infty} u_5(t) \sin \pi n z \]  
(34)

Where

\[ u_5(t) = \frac{1}{q_{11}} \left(1 - e^{-q_1 t}\right) + \sum_{n=1}^{\infty} q_{25} p_4 \left(\frac{1}{q_{11}} \left(1 - e^{-q_1 t}\right) - te^{-q_1 t}\right) + \sum_{n=1}^{\infty} q_{28} \left(\frac{1}{q_{11}} \left(1 - e^{-q_1 t}\right) - te^{-q_1 t}\right) - \frac{1}{q_{11} - q_0} \left(e^{-q_0 t} - e^{-q_1 t}\right) - \frac{1}{q_0} \left(e^{q_0 t} - e^{-q_1 t}\right) \]

\[ \sum_{n=1}^{\infty} \left(\frac{1}{q_{11}} \left(1 - e^{-q_1 t}\right) - \frac{1}{q_{11} - q_0} \left(e^{-q_0 t} - e^{-q_1 t}\right) \right) + \sum_{n=1}^{\infty} q_{29} q_{11} \left(\frac{1}{q_{11}} \left(1 - e^{-q_1 t}\right) - \frac{1}{q_{11} - q_0} \left(e^{-q_0 t} - e^{-q_1 t}\right) \right) + \sum_{n=1}^{\infty} q_{35} q_{11} \left(\frac{1}{q_{11}} \left(1 - e^{-q_1 t}\right) - \frac{1}{q_{11} - q_2} \left(e^{-q_2 t} - e^{-q_1 t}\right) \right) + \sum_{n=1}^{\infty} q_{45} \left(\frac{1}{q_{11}} \left(1 - e^{-q_1 t}\right) - \frac{1}{q_{11} - q_2} \left(e^{-q_2 t} - e^{-q_1 t}\right) \right) + \sum_{n=1}^{\infty} q_{55} \left(\frac{1}{q_{11} - q_0} \left(e^{-q_0 t} - e^{-q_1 t}\right) - \frac{1}{q_{11} - q_2} \left(e^{-q_2 t} - e^{-q_1 t}\right) \right) - \sum_{n=1}^{\infty} q_{65} \left(\frac{1}{q_{11} - q_0} \left(e^{-q_0 t} - e^{-q_1 t}\right) - \frac{1}{q_{11} - q_2} \left(e^{-q_2 t} - e^{-q_1 t}\right) \right) \]
\[ \theta_{1}(z,t) = \sum_{n=1}^{\infty} u_{n}(t) \sin \pi nz \]  

Where

\[ u_{n}(t) = \sum_{n=1}^{\infty} q_{30} \left( \frac{1}{q_{0}} (1 - e^{-q_{0}t}) - te^{-q_{0}t} \right) + \sum_{n=1}^{\infty} q_{1} q_{31} \left( \frac{1}{q_{0}} (1 - e^{-q_{1}t}) - 2te^{-q_{1}t} - \frac{1}{q_{0}} (e^{-2q_{0}t} - e^{-q_{1}t}) \right) + \]

\[ \begin{align*}
q_{32} \frac{1}{q_{0}} (1 - e^{-q_{0}t}) + \sum_{n=1}^{\infty} q_{1} q_{33} \left( \frac{1}{q_{0} - 2q_{11}} (e^{-q_{11}t} - e^{-q_{0}t}) \right) + \\
\sum_{n=1}^{\infty} \sum_{n=1}^{\infty} q_{35}^{2} q_{36} \left( \frac{1}{q_{0}} (1 - e^{-q_{0}t}) - \frac{2}{q_{0} - q_{11}} (e^{-q_{11}t} - e^{-q_{0}t}) \right) + \frac{1}{q_{0} - 2q_{11}} (e^{-2q_{11}t} - e^{-q_{0}t}) \end{align*} \]

\[ w_{1}(z,t) = \sum_{n=1}^{\infty} u_{n}(t) \sin \pi nz \]  

Where

\[ u_{n}(t) = \sum_{n=1}^{\infty} q_{36} \left( te^{-q_{0}t} \right) + \sum_{n=1}^{\infty} \sum_{n=1}^{\infty} q_{37} q_{36} \left( te^{-q_{11}t} + \frac{1}{q_{0}} (e^{-q_{0}t} - e^{-q_{11}t}) \right) + q_{38} \left( \frac{1}{q_{11}} (1 - e^{-q_{11}t}) \right) + \]

\[ \sum_{n=1}^{\infty} q_{34} P_{4} \left( \frac{1}{q_{11}} (1 - e^{-q_{11}t}) + te^{-q_{11}t} \right) \]

\[ \phi_{1}(z,t) = \sum_{n=1}^{\infty} \mu_{n}(t) \sin \pi nz \]  

Where
Therefore the solutions to the governing equations are given as:

\[
\theta(z,t) = z^+ \sum_{n=1}^{\infty} q_1 \left(1 - e^{-q_1}\right) \sin n\pi z + \sum_{n=1}^{\infty} u_n(t) \sin n\pi z
\]
\[ \phi(z,t) = z + v_2(z,t) + S \sum_{n=1}^{\infty} u_n(t) \sin n\pi z \] (39)

\[ w(z,t) = \sum_{n=1}^{\infty} \frac{q_0}{q_{1n}} e^{-\theta_{1n} t} \sin n\pi z + S \sum_{n=1}^{\infty} u_n(t) \sin n\pi z \] (40)

\[ u(z,t) = z + \sum_{n=1}^{\infty} \frac{q_1}{q_{1n}} (1-e^{-\theta_{1n} t}) \sin n\pi z + S \sum_{n=1}^{\infty} u_n(t) \sin n\pi z \] (41)

4 Results and Discussion

The system of partial differential equations describing unsteady couette flow of an electrically conducting incompressible fluid bounded by two parallel non conducting porous plates is solved analytically using eigenfunction expansion method. The analytical solutions of the governing equations are computed and presented graphically with the aid of a computer symbolic algebraic package MAPLE 17 for the values of the following parameters:

Re=1, Ra=1, S=0.1, Pr=0.71, \( \theta_{1n} = 1 \), \( K_r = 0.5 \), \( S_c = 0.22 \)

Bi=1, Be=1, \( a = 0.1 \), \( c = 0.2 \), \( P = 1 \), \( T_D = 0 \), \( E_c = 0.01 \)

\( Gr_\phi = 0.2 \), \( Gr_\phi = 0.2 \), \( \sigma = 2 \)

The Figs. 2-12 Explains the graphs of primary and secondary velocities, temperature and concentration against different dimensionless parameters.

![Graph](image)

Fig. 2. Relationship between primary velocity and time for different values of re
Fig. 2 presents the graph of primary velocity with time for different values Reynolds number (Re). It is observed that primary velocity increases with time and also increases as Reynolds number increases.

Fig. 3 presents the graph of concentration profile with time $t$ for different values of Reynolds number. It is observed that the concentration profile increases with time and also, increases as Reynolds number increases.

Fig. 4 shows the influence of radiation on the primary velocity profile. It is evident that the primary velocity increases with time. Also, an increase in the radiation parameter is found to decelerate the primary velocity of the flow.
Fig. 5. Relationship between temperature and time for different values of Ra

Fig. 5 displays the effect of the thermal radiation parameter on the thermal profile of the flow with time t. It is observed that the flow field suffers a decrease in temperature as radiation parameter increases while as radiation parameter increases the temperature decreases with time t.

Fig. 6. Relationship between concentration and time for different values of Ra

Fig. 6 depicts the graph of concentration with time t for different values of radiation parameter. It is evident that concentration increases with time and also increases as radiation increases.

Fig. 7 illustrates the graph of temperature with time for different values of suction parameter. It is seen that temperature decreases with time and also decreases as suction parameter increases.

Fig. 8 presents the effect of the suction parameter on concentration along distance y. It is observed that an increase in suction parameter leads to decrease in concentration while concentration is observed to increase along distance y.
Fig. 7. Relationship between temperature and time for different values of S

Fig. 8. Relationship between concentration and distance for different values of S

Fig. 9. Relationship between primary velocity and time for different values of σ
Fig. 9 shows the influence of pressure gradient on the primary velocity with time. It is observed that an increase in pressure gradient leads to an increase in primary velocity. Also, primary velocity is found to increase with time.

![Graph of primary velocity with time for different pressure gradients](image1)

**Fig. 10. Relationship between secondary velocity and time for different values of $\sigma$**

Fig. 10 shows the effect of pressure gradient on secondary velocity with time $t$. It is observed that increase in pressure gradient leads to decrease in secondary velocity of the fluid while the secondary velocity is observed to decrease with time $t$.

![Graph of secondary velocity with time for different values of $\sigma$](image2)

**Fig. 11. Relationship between secondary velocity and time for different values of $Ha^2$**

Fig. 11 presents the graph of secondary velocity with time $t$ for different values of Hartman number. It is observed that increase in Hartman number leads to decrease in secondary velocity. This is due to the retarding Lorentz force which acts in opposite direction of the fluid flow when magnetic field is applied. This type of resisting force, slows down the velocity as shown in the figure.
Fig. 12. Relationship between temperature and time for different values of $Ha^2$

Fig. 12 shows the effect of Hartman number with time $t$ on the temperature profile. It is observed that increase in Hartman number leads to a decrease in temperature. Also, the temperature profile is observed to decrease with time.

5 Conclusion

For constant pressure gradient, the unsteady MHD Couette flow through a porous medium of a viscous incompressible fluid bounded by two parallel porous plates under the influence of thermal radiation and chemical reaction is investigated. A uniform suction and injection are applied perpendicular to the plate. The transformed conservation equations are solved analytically subject to physically appropriate boundary conditions by using the Eigenfunction expansion technique. From the results obtained, we can conclude that:

1. The increase in Hartman number leads to decrease in velocity. This is due to the retarding Lorentz force which acts in opposite direction of the fluid flow when magnetic field is applied.
2. Concentration profile increases with time and also, increases as Reynolds number increases.
3. The increase in the radiation parameter is found to decelerate the velocity of the flow.
4. The flow field suffers a decrease in temperature as the radiation parameter increases while as radiation parameter the temperature profile is observed to decreases with time $t$.

Competing Interests

Authors have declared that no competing interests exist.

References


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