Analysis of Inventory Model with Time Dependent Quadratic Demand Function Including Time Variable Deterioration Rate without Shortage

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Authors’ contributions

This work was carried out in collaboration between both authors. Author MAR designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author MFU managed the analyses of the study. Authors MAR and MFU both managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

This paper deals with an inventory model for deteriorating items along with time dependent demand which is quadratic function of time. In this model, the deterioration rate follows deterministic deterioration which is quadratic function of time. Here shortages are not allowed. The main purpose of this paper is to investigate minimum total cost per unit time of the inventory model. The result are validated with the help of numerical example. The sensitivity analysis of the optimal solution with respect to various parameters is carried out. Finally the behavior of the relation between parameters and total inventory cost have been shown using figure.

Keywords: Inventory; deterministic inventory model; deterioration items; sensitivity analysis.

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1 Introduction

Inventory is one of the predominant assets of a business organization. Inventory is mandatory for each business organization to ensure smooth operation of the production process, to decrease the ordering cost of inventory, to take benefits of quantity discount, to utilize and optimize the plant capacity. Inventory management is an important scientific device for controlling inventory and eliminating wastage, is considered an integral part of Industrial management in modern times. The main goal of inventory management is to hold inventories at the least possible cost, given the objectives to confirm uninterrupted supplies for ongoing operations. When making decisions on inventory, management has to find a compromise between the different cost components, such as the costs of supplying inventory, inventory-holding costs and costs resulting from insufficient inventories.

In the study of inventory study, deterioration of products and demand considers an important part of the research. The effect of deterioration is very important in inventory system. Generally, deterioration is defined as decay or damage such that the item cannot be used for its original purpose. According to the study of Wee in 1993 [1], deteriorating items refers to the items that become decayed, damaged, evaporative, expired, devaluation and so on through time. Based on Wee [1] definition, deteriorating items can be categorized two ways. The first category refers to the items that become decayed, damaged, and evaporative or expired through time like meat, vegetables, fruit, and flowers and so on. On the other hand, the second category refers to the items that lose partially or entire value through time because of new technology like computer chips, mobile phone, and fashion items and so on. The inventory system of deteriorating items was first observed by Whitin [2], He studied fashion items deteriorating at the end of the shortage period. Then Ghare and Schrader [3], extended in their observation that the consumption of the deteriorating items was closely relative to an exponential function of time. They introduced the deteriorating items inventory system as stated below:

\[ \frac{dI(t)}{dt} + \theta I(t) = -D(t) \]

In the above equation, \( \theta \) means the deteriorating rate of item, \( I(t) \) refers to the inventory level at time \( t \) and \( D(t) \) is the rate of change of demand due to time \( t \).

Demand is a key factor in an inventory system. A relation showing the quantities of a goods that consumers are willing and able to buy at various prices per period. Demand is the rate at which consumers want to buy a product. Economic theory describes that demand consists of two factors such as taste of the products and ability to buy. Taste means the preference of goods what he wants to get at a specific price. On the other hand, ability is the indicator for which one can buy goods at certain range. Both factors of demand depend on the market price. When the market price for a product is high, the demand will be low. When price is low, demand is high. At very low prices, many consumers will be able to purchase a product. Most of the researchers have concentrated their attention on a time dependent demand function. In general, constant demand rate should not consider for each types of items like fashionable cloths, computer equipment’s, etc. Because the demand function of this type of items is totally dependent on time. A group of researchers is introduced the demand as a linear function of time. Linear time dependent demand function indicates a uniform change in the demand rate of product per unit time. This idea is not realistic for every moment in the market situation. On the other hand, an exponential time dependent demand also seems to be unrealistic. Generally, exponential rate of change is very high. For this reason, the market demand of any product may not undergo such a high rate of change as exponential. Donaldson [4], was first to analysis about the inventory replenishment strategy for a linear trend in demand function. Kalpakam, and Shanthi [5], who analyzed perishable system with modified base stock policy and random supply quantity. Dye, C.-Y., Chang and Teng [6], who studied deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging. Then Singh and Pattanayak [7], who introduced an EOQ model for deteriorating items that deteriorates according to variable deterioration rate with demand function being linear with partial backlogging. Peter et al [8], have proposed an inventory model where the demand rate is ramp type. Mohan

In this paper, we have introduced an inventory model by considering the quadratic demand function of time. Basically, quadratic function of time indicates the steady growth or decline of the demand in the best way. Also we have considered time dependent deterioration rate. In real situation, deterioration rate cannot be constant. Due to various causes deterioration rate can be changed. The proposed inventory model is based on deteriorating items like various fruits and vegetables whose deterioration rate will be increased with respect of time. In our model shortages are not allowed. We have solved the model to optimize the total cost which is to be minimized. A numerical example is analyzed to check the validity of the model. Also the sensitivity of different parameter is observed graphically.

2 Assumptions

To develop the proposed mathematical model of the inventory system, the following assumptions are considered in this paper:

I. The inventory system involves single type of items.
II. Replenishment rate is infinite, i.e. Replenishment rate is instantaneous.
III. The demand rate of the item is considered by a quadratic and continuous function of time. $D(t)$ is the time dependent demand function which is defined by

$$D(t) = a + bt + ct^2,$$

where $a, b, c > 0$. Here $a$ is the initial rate of demand, $b$ is the rate at which the demand rate increases and $c$ is the rate at which the change in the demand rate itself increases.

IV. The deterioration rate is variable rate of deterioration on the on-hand inventory per unit time and there is no repair or replenishment of the deteriorated items within the cycle.

Variable rate of deterioration of an item where $\theta(t) = \theta t^2; 0 < \theta < 1$.

V. Shortages is not allowed.
VI. There is no provision for repair or replacement of deteriorated units.
VII. Time horizon is infinite.

3 Notations

$T$ The length of replenishment cycle in traditional system (per year)
$I(t)$ The inventory level at time $t$ ($0 < t < T$)
$A$ The ordering cost of inventory per order ($TK/order$)
$h$ The unit holding cost per unit time
$p$ The purchase cost per unit of item
$D(t)$ Demand rate of time which is defined by

$$D(t) = a + bt + ct^2, a > 0, b \neq 0 & c \neq 0.$$ Here $a$ is the initial rate of demand, $b$ is the rate with which the demand rate increases and the rate of change in the demand rate itself changes at a rate $c$.

$\theta(t)$ The variable rate of deterioration of an item where $\theta(t) = \theta t^2; 0 < \theta < 1$

$IM$ The maximum inventory level during $[0,T]$

$T(C)$ The average cost per unit time.
4 Mathematical Formulation

This inventory model is developed by the consideration of the replenishment problem of a single non-instantaneous deteriorating item without shortage. The inventory model runs as follows:

\[ \frac{dI(t)}{dt} + \theta(t)I(t) = -(a + bt + ct^2) \]  

(1)

with conditions \( 0 \leq t \leq T, I(0) = lM, I(t) = 0 \) if \( t = T \).

5 Analytic Solution of the Model

The inventory depletes during the period \([0, T]\) due to the deterioration and demand. The inventory level at any time during \([0, T]\) is described by equation (1) which is a first order and first degree ordinary differential equation.

Integrating factor \( I.F = e^{\int \theta t^2 dt} = e^{\frac{\theta t^3}{3}} \)

Therefore \( I(t) e^{\frac{\theta t^3}{3}} = -\int (a + bt + ct^2) e^{\frac{\theta t^3}{3}} dt \)

\[ = -a \left( t + \frac{\theta t^4}{12} \right) - b \left( \frac{t^2}{2} + \frac{\theta t^5}{15} \right) - c \left( \frac{t^3}{3} + \frac{\theta t^6}{18} \right) + W \]  

(3)

\( I(t) = 0 \) if \( t = T \) in equation (3), we have \( W = a \left( t + \frac{\theta t^4}{12} \right) + b \left( \frac{t^2}{2} + \frac{\theta t^5}{15} \right) + c \left( \frac{t^3}{3} + \frac{\theta t^6}{18} \right) \)
Therefore \( I(t) = a \left[ (T - t) + \frac{\theta}{12} (T^4 - t^4) \right] - a \left[ \frac{\theta}{3} (T^3 - t^3) + \frac{\theta^2}{36} (T^4 - t^4) \right] \)

\[
+ b \left[ \frac{1}{2} (T^2 - t^2) + \frac{\theta}{15} (T^5 - t^5) \right] - b \left[ \frac{\theta}{6} (T^3 - t^3) + \frac{\theta^2}{45} (T^5 - t^5) \right]
\]

\[
+ c \left[ \frac{1}{3} (T^3 - t^3) \right] - c \left[ \frac{\theta}{6} (T^3 - t^3) + \frac{\theta^2}{36} (T^4 - t^4) \right] + \frac{1}{2} \left[ \frac{\theta}{12} (T^4 - t^4) \right] + \frac{1}{3} (T^3 - t^3) - \frac{1}{2} (T^2 - t^2) + \frac{\theta}{15} (T^5 - t^5) - \frac{\theta}{6} (T^3 - t^3) - \frac{\theta^2}{45} (T^5 - t^5) \right]
\]

\[
+ c \left[ \frac{1}{3} (T^3 - t^3) \right] - c \left[ \frac{\theta}{6} (T^3 - t^3) + \frac{\theta^2}{36} (T^4 - t^4) \right] \quad (4)
\]

We have, Inventory holding cost per cycle, \( IHC = h \int_0^T I(t) \, dt \)

\[
= h \left( \frac{a \theta^2}{2} + \frac{a \theta^3}{20} - \frac{a \theta^2 t^3}{288} + \frac{b \theta^3}{3} + \frac{b \theta^3 t}{24} - \frac{b \theta^3 t^3}{324} + \frac{c \theta^3 t^3}{4} + \frac{c \theta^3 t^3}{28} - \frac{c \theta^3 t^3}{360} \right) \quad (5)
\]

Now, the total demand of the cycle during \([0, T]\) is given by

\[
\int_0^T (a + bt + ct^2) \, dt = T(a + \frac{bt}{2} + \frac{ct^2}{3}) \quad (6)
\]

Therefore, the number of deteriorating unit is

\[
W - \int_0^T D(t) \, dt = a \left( t + \frac{\theta t^4}{12} \right) + b \left( \frac{t^2}{2} + \frac{\theta t^5}{15} \right) + c \left( \frac{t^3}{3} + \frac{\theta t^6}{18} \right) - T
\]

\[
= \frac{\theta t^3}{3} \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) \quad (7)
\]

Hence, deterioration rate per cycle

\[
D_c = \text{purchase cost per cycle } \times \text{number of deteriorated units}
\]

\[
= \frac{p \theta t^3}{3} \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right)
\]

At the initial stage i.e. at \( t = 0 \), \( I(0) = IM \).

\[
IM = T \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) + \frac{\theta t^4}{3} \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) \quad (8)
\]

We have, ordering size \( O_s = IM + IB \)

\[
= T \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) + \frac{\theta t^4}{3} \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) + 0
\]

\[
= T \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) + \frac{\theta t^4}{3} \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) \quad (9)
\]

Also, Purchase cost \( P_c = p \times O_s \)

\[
= p \left[ T \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) + \frac{\theta t^4}{3} \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) \right]
\]

Therefore, total cost per unit time (TC) is given by
\[ TC = \frac{\text{Ordering cost} + \text{Holding cost} + \text{Deterioration cost} + \text{Purchase cost}}{T} \]

\[ = \frac{1}{T} \left[ A + \frac{haT^2}{2} \left( 1 + \frac{\theta T^3}{10} - \frac{\theta^2 T^6}{144} \right) + \frac{hbT^3}{3} \left( 1 + \frac{\theta T^3}{8} - \frac{\theta^2 T^6}{108} \right) + \frac{hcT^4}{4} \left( 1 + \frac{\theta T^3}{7} - \frac{\theta^2 T^6}{90} \right) + pT \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) \right. \]

\[ + \left. \frac{2p\theta T^4}{3} \left( \frac{a}{4} + \frac{bt}{5} + \frac{ct^2}{6} \right) \right] \]

To get the optimal solution we need to solve the following equations

\[ \frac{dT(C)}{dT} = 0 \text{ and } \frac{d^2T(C)}{dT^2} > 0. \]

Now

\[ \frac{dT(C)}{dT} = \frac{1}{T} \left[ h \left( 1 + \frac{\theta T^3}{4} - \frac{\theta^2 T^6}{36} \right) (aT + bT^2 + cT^3) + \left( \frac{2p\theta T^3}{3} + p \right) (a + bT + cT^2) \right] \]

\[ - \frac{1}{T^2} \left[ A + \frac{haT^2}{2} \left( 1 + \frac{\theta T^3}{10} - \frac{\theta^2 T^6}{144} \right) + \frac{hbT^3}{3} \left( 1 + \frac{\theta T^3}{8} - \frac{\theta^2 T^6}{108} \right) + \frac{hcT^4}{4} \left( 1 + \frac{\theta T^3}{7} - \frac{\theta^2 T^6}{90} \right) + pT \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) \right. \]

\[ + \left. \frac{2p\theta T^4}{3} \left( \frac{a}{4} + \frac{bt}{5} + \frac{ct^2}{6} \right) \right] \]

\[ \frac{d^2T(C)}{dT^2} = \frac{1}{T} \left[ h \left( 1 + \frac{\theta T^3}{4} - \frac{\theta^2 T^6}{36} \right) (a + 2bT + 3cT^2) + h(aT + bT^2 + cT^3) \right] \left( \frac{3\theta T^2}{4} - \frac{\theta^2 T^5}{6} \right) \]

\[ + \left( \frac{2p\theta T^3}{3} + p \right) \left( b + 2cT \right) + 2p\theta T^2 (a + bT + cT^2) \]

\[ - \frac{1}{T^2} \left[ h \left( 1 + \frac{\theta T^3}{4} - \frac{\theta^2 T^6}{36} \right) (aT + bT^2 + cT^3) + \left( \frac{2p\theta T^3}{3} \right) (a + bT + cT^2) \right] \]

\[ - \frac{1}{T^2} \left[ h \left( 1 + \frac{\theta T^3}{4} - \frac{\theta^2 T^6}{36} \right) (a + 2bT^2 + cT^3) + \frac{2p\theta T^3}{3} \left( a + \frac{b}{5} + \frac{4bT}{5} + \frac{cT}{3} + \frac{2cT^2}{3} \right) \right] \]

\[ + \left( \frac{bt}{3} + \frac{cT^2}{3} \right) \left( \frac{1}{T} \left[ A + \frac{haT^2}{2} \left( 1 + \frac{\theta T^3}{10} - \frac{\theta^2 T^6}{144} \right) \right. \right] \]

\[ + \frac{hbT^3}{3} \left( 1 + \frac{\theta T^3}{8} - \frac{\theta^2 T^6}{108} \right) + \frac{hcT^4}{4} \left( 1 + \frac{\theta T^3}{7} - \frac{\theta^2 T^6}{90} \right) + pT \left( a + \frac{bt}{2} + \frac{ct^2}{3} \right) \]

\[ + \left. \frac{2p\theta T^4}{3} \left( \frac{a}{4} + \frac{bt}{5} + \frac{ct^2}{6} \right) \right] \]

6 Results Discussion and Computational Analysis

In this section, a numerical example is considered to illustrate this maintenance model. Parameter numerical example has been considered to check the validity inventory model in proper units:

\[ A = 12 \frac{T}{\text{order}}, \quad a = 10, \quad b = 8, \quad c = 5, \quad h = \frac{17k}{\text{kg}}, \quad p = 15Tk/kg, \quad \theta = 87\% \]

Solving the equation and it has been obtained optimum value of \( T = 1.4790 \) and the minimum average cost per unit time is evaluated \( TC = 46.0117 \).
7 Sensitivity Analysis

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. Here, the sensitivity analysis for total cost per unit time TC is carried out with respect to the changes in the values of the parameters of quadratic function, unit holding cost per unit time h, ordering cost A, Holding cost h, purchase cost p, deteriorating cost, parameters a, b & c. The sensitivity analysis is performed by considering variation in each one of the above parameters by 5% change in stipulated standard value, keeping all other remaining parameters as fixed.

8 Sensitivity of Different Parameters with Total Cost Per Unit Time

### Table 1. (Sensitivity of ordering cost A)

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Table 5. (Sensitivity of holding cost $h$)

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<td>47.1038</td>
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<tr>
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<tr>
<td>13</td>
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<td>52.5641</td>
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Table 6. (Sensitivity of deterioration rate $\theta$)

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<th>Parameter Value</th>
<th>TC</th>
</tr>
</thead>
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<td>1</td>
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<td>43.0083</td>
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<td>2</td>
<td>0.6525</td>
<td>43.5237</td>
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<td>3</td>
<td>0.696</td>
<td>44.0332</td>
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<tr>
<td>4</td>
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<td>44.5368</td>
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<tr>
<td>5</td>
<td>0.783</td>
<td>45.0344</td>
</tr>
</tbody>
</table>
Index | Parameter Value | TC  
---|---|---  
6 | 0.8265 | 45.526  
7 | 0.8700 | 46.0117  
8 | 0.9135 | 46.4915  
9 | 0.9570 | 46.9653  
10 | 1.0005 | 47.4331  
11 | 1.0440 | 47.895  
12 | 1.0875 | 48.3509  
13 | 1.1310 | 48.8009  

Table 7. (Sensitivity of purchase cost p)  
Index | Parameter Value | TC  
---|---|---  
1 | 0.35 | 41.1947  
2 | 0.375 | 41.9975  
3 | 0.4 | 42.8004  
4 | 0.425 | 43.6032  
5 | 0.45 | 44.406  
6 | 0.475 | 45.2089  
7 | 0.5 | 45.9117  
8 | 0.5250 | 46.8146  
9 | 0.5500 | 47.6174  
10 | 0.5750 | 48.4203  
11 | 0.6 | 49.2231  
12 | 0.6250 | 50.0259  
13 | 0.6500 | 50.8288  

9 Graphical Presentation for the Effects of Parameters on Total Cost of Per Unit Time

Fig. 2. Effects of ordering cost on total cost per unit time
Comments: From the Fig. (2), it is observed that due to increase or decrease of ordering cost A per unit, the total cost per unit time is increasing. Similarly due to decrease of ordering cost A per unit, the total cost per unit time is decreasing. Hence total cost per unit time is affected by the changes of ordering cost.
**Fig. 3. Effects of parameter a on total cost per unit time**

*Comments:* From the Fig. (3), it is analyzed that when the values of parameter \( a \) increase per unit then total cost per unit time increases as well as when the values of parameter \( a \) decrease per unit then total cost per unit time decreases. Hence total cost per unit time is influenced by the changes of parameter \( a \).

**Fig. 4. Effects of parameter b on total cost per unit time**

*Comments:* From the Fig. (4), it is examined that when the values of parameter \( a \) increase per unit then total cost per unit time increases. Also when the values of parameter \( a \) decrease per unit then total cost per unit time decreases. Hence total cost per unit time is moved by the changes of parameter \( b \).

**Fig. 5. Effects of parameter c on total cost per unit time**

*Comments:* From the Fig. (5), it is inspected that when the values of parameter \( a \) increase per unit then total cost per unit time increases as well as when the values of parameter \( a \) decrease per unit then total cost per unit time decreases. Hence total cost per unit time is influenced by the changes of parameter \( c \).
Fig. 6. Effects of holding cost on total cost per unit time
Comments: From the Fig. (6), it is viewed that when the values of holding cost $h$ increase per unit then total cost per unit time increases. On the other hand, the values of holding cost $h$ decrease per unit then total cost per unit time decreases. Hence total cost per unit time is affected by the changes of holding cost $h$.

Fig. 7. Effects of deterioration rate on total cost per unit time
Comments: From the Fig. (7), it is remarked that due to increase or decrease of deterioration rate $\theta$ per unit, the total cost per unit time is increasing. Similarly due to decrease of deterioration rate $\theta$ per unit, the total cost per unit time is decreasing. Hence total cost per unit time is affected by the changes of deterioration rate.

Fig. 8. Effects of purchase cost on total cost per unit time
Comments: From the Fig. (8), it is noticed that on account of increasing or decreasing of purchase cost $p$ per unit, the total cost per unit time is increasing. Similarly due to decrease of purchase cost $p$ per unit, the total cost per unit time is decreasing. Hence total cost per unit time is affected by the changes of purchase cost.
10 Conclusion

The inventory model has been upheld with deterioration rate and without shortages. This model can apply in any industry to determine the effect of cost per unit time for variation of different parameters. It is evident that

- When the values of parameters of quadratic demand function increase then the total cost per unit time increase as well as when the values of parameters of quadratic demand function decrease then the total cost per unit time decrease.
- When the values of ordering cost, holding cost and purchase cost per unit time increase then the total cost per unit time increase as well as when the values of ordering cost, holding cost and purchase cost per unit time decrease then the total cost per unit time decrease.
- When the deterioration rate increase then the total cost per unit time increase rapidly whereas when the deterioration decrease then the total cost per unit time decrease rapidly.

Competing Interests

Authors have declared that no competing interests exist.

References


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