On the Efficiency of Modified Regression-type Estimators Using Robust Regression Slopes and Non-conventional Measures of Dispersion

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Authors’ contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

Supplementary variables associated with the study variables have been identified to be helpful in improving the efficiency of ratio, product and regression estimators both at planning and estimation stages. The existing regression-based estimators are functions of regression slopes and known auxiliary variables which are sensitive to outliers. Zaman & Bulut [1] and Zaman [2] addressed the issue of regression slopes in the aforementioned estimators using robust regression slopes like Huber-M, Hampel-M, Least Trimmed Squares (LTS) and Least Absolute Deviation (LAD). However, their estimators still utilized known auxiliary functions which are also sensitive to outliers or extreme values. Similarly, Yadav and Zaman [3] suggested non-conventional robust parameters of auxiliary variable which are robust against outliers. However, the problems of effects of outliers on regression slopes were not considered. In this study, the estimators of Zaman & Bulut [1] and Zaman [2] estimators were modified using robust non-conventional measures and Yadav and Zaman [3] estimators were modified using robust regression slopes. The properties (Biases and MSEs) of the modified estimators were derived up to the first order of approximation using Taylor series approach. The efficiency conditions of the proposed estimators over the existing estimators considered in the study were established. The empirical studies were conducted using stimulation data to investigate the efficiency of the proposed estimators over the efficiency of the existing estimators. The results revealed that the proposed...
estimators have minimum MSEs and higher PREs among all the competing estimators for the simulated populations in the study. This implies that the proposed estimators are more efficient and can produce better estimate of the population mean compared to other existing estimators considered in the study.

Keywords: Outliers; estimators; robust measures; auxiliary variable; population mean.

1 Introduction

The use of supplementary (auxiliary) information has been widely discussed in sampling theory. Auxiliary variables are used in survey sampling to obtain improved sampling designs and to achieve higher precision in the estimates of some population parameters such as the mean or the variance of the variable under study. This information may be used at both designing (e.g., stratification, systematic or probability proportional to size sampling designs) and estimation stages. In survey sampling literature, variety of techniques which utilized auxiliary information by means of ratio, product and regression methods have proposed. Similarly, variety of estimators have been proposed by linking together ratio, product or regression estimators, each one exploiting a number of known parameters of the auxiliary variable such as the coefficient of variation \( C \), the coefficient of skewness \( \beta_2(x) \), standard deviation \( \delta \), the coefficient of skewness \( \beta_1(x) \), the correlation coefficient between the study variable and an auxiliary variable \( \rho_{xy} \). Sisodia and Dwivedi [5] suggested a modified ratio estimator using the coefficient of variation \( C \) of an auxiliary variable X for estimating the population mean \( \bar{Y} \). Upadhyaya and Singh [6] suggested another modified ratio estimator using a linear combination of the coefficient of variation \( C \) and coefficient of the kurtosis \( \beta_2(x) \). Singh and Tailor [7-9] proposed another estimator using the correlation coefficient \( \rho_{xy} \) between X and Y. By using the population variance \( S^2 \) of an auxiliary variable X. Singh [7-9] proposed another modified ratio estimator. Also, Singh [7-9] used a linear combination of the coefficient of the kurtosis \( \beta_2(x) \), standard deviation \( \delta \), and coefficient of skewness \( \beta_1(x) \) for estimating the population mean of the study variable \( \bar{Y} \). Motivated by Singh [7-9], Yan and Tian [10] used a linear combination of the coefficient of kurtosis \( \beta_2(x) \), coefficient of skewness \( \beta_1(x) \), and coefficient of variation \( C \) of the auxiliary variable X. More recently, Subramani and Kumarapandiyam [11] suggested a new modified ratio estimator using known population median \( M_d \) of an auxiliary variable. Subramani and Kumarapandiyam [11,12] have also suggested modified ratio estimators using the known median and the coefficient of kurtosis, median and coefficient of skewness, median and the coefficient of variation, and median and the coefficient of correlation. Other authors that had work in this direction include Hartley-Ross (1954), Quenouille’s [13], Singh [14,15], Naik and Gupta [16], Kadilar and Cingi [17], Singh and Espejo [7-9], Shabbir and Yaah [18], Abu-Dayeh et al. [19], Kadilar and Cingi [20], Jhajj et al. [21], Khoshnevisan et al. [22], Kadilar and Cingi [23], Perri [24], Singh et al. [25], Gupta and Shabbir [26], Sharma and Taylor [27], Grover and Kaur [28], Subramani and Kumarapandiyam [11,12,29,30], Singh and Kumar [31], Nadia and Mohammad [32], Audu and Singh [33], Tailor et al. [34], Lu [35], Sharma and Singh [36], Lu and Yan [37], Verma et al. [38], Audu and Adewara [39,40], Muili and Audu [41], Muili et al. [42,43,44], Singh et al. [45], Audu et al. [46-48], Audu et al. [49-53], Audu et al. [54-56,57], Yunusa et al. [58], Zaman et al. [59], Audu and Singh [60], Zakari et al. [61,62]

Regression estimator is used to estimate the population characteristics such as population mean, total and variance when the regression line of y on x does not pass through the origin but makes an intercept along the y-axis. Many modifications of the regression type estimators have been done to improve their efficiency by using a number of known parameters of the auxiliary variable such as the coefficient of variation \( C \), the coefficient of kurtosis \( \beta_2(x) \), standard deviation \( \delta \), the coefficient of skewness \( \beta_1(x) \), and the correlation coefficient.
between the study variable and an auxiliary variable $Y$. Shabbir & Gupta [63] proposed a regression ratio type exponential estimator by combining Rao’s [64] and Bedi’s [65] estimators. Following these works, Grover & Kaur [28] introduced a regression exponential type estimator. Ozgul & Cingi [66] proposed a new class of exponential regression cum ratio estimator using functions of any known population parameters of the auxiliary variable, such as standard deviation, coefficient of variation, coefficient of skewness, coefficient of kurtosis and coefficient of correlation of the auxiliary variable for the estimation of finite population mean. Several authors like Kadilar and Cingi [67], Kadilar and Cingi [68], Subramani and Kumarapandiyam (2006), Abid et al. [69], Subzar et al. [70], Subzar et al. [71], Subzar et al. [72], Subzar et al. [73] have proposed some regression-based estimators which utilized known functions of auxiliary variables. However, these auxiliary parameters are sensitive to outliers or extreme values that do present in the population distributions. Outliers are observations that are distant from other observations in the population. They tend to inflate average deviation of the entire observations from central values. When there are outliers in data, the auxiliary functions like Kurtosis, Skewness, Coefficient of variation, standard deviation etc, will be affected and consequently the efficiency of the estimators which utilize these functions will drastically reduce. Some of the regression estimators in the above paragraph are function of these auxiliary functions. Similarly, regression slope in the regression estimators is also sensitive to outliers and its effect will decreases the efficiency of the estimators. Zaman & Bulut [1], Zaman [2] suggested robust regression slopes like Hampel M, Huber M, LTS and LAD methods which are robust against outliers as an alternative to regression slope in the regression estimators of the previous authors. However, the problem of effects of outliers on auxiliary functions in the previous studies was not addressed. Similarly, Yadav and Zaman [3] suggested non-conventional robust parameters of auxiliary variable which are robust against outliers. However, the problems of effects of outliers on regression slopes were not considered. Recently, the concept of using robust regression slopes has been extended to modification of variance estimators, estimation in stratified and double sampling schemes etc (See Bulut and Zaman [74], Zaman and Bulut [75], Zaman and Bulut [76,77], Zaman and Bulut [78]. This current study focused on the modification of robust regression estimators using robust non-conventional measures (Gini’s mean, Downton’s method, and probability weighted moment) and robust regression slopes simultaneously to address the effect of outliers on auxiliary functions and regression slopes respectively.

### 1.1 Symbols and Notations

Under this section various notations have been shown which have been used throughout the study.

- **N**: Population Size
- **n**: Sample Size
- **f = n/N**: Sampling fraction
- **Y**: Study Variable (Primary variable)
- **X**: Auxiliary Variable (Secondary variable)

\[
R = \frac{\bar{Y}}{\bar{X}}
\]  
- Ratio of Population mean of Y to Population of X

\[
\bar{Y} = \frac{1}{N} \sum_{i=1}^{N} Y_i
\]  
- Population Mean of Study Variable Y

\[
\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i
\]  
- Population Mean of Auxiliary Variable X

\[
S_y = \sqrt{\frac{\sum_{i=1}^{N} (y_i - \bar{Y})^2}{(N-1)}}
\]  
- Population Standard Deviation of Study Variable Y
\[
S_x = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{X})^2}{(N-1)}} 
\]
- Population Standard Deviation of Auxiliary Variable X

\[
\rho_{yx} = \frac{S_{yx}}{S_x S_y} \]
- Population Correlation Coefficient between Study and Auxiliary Variables

\[
S_{yx} = \frac{\sum_{i=1}^{N} (y_i - \bar{Y})(x_i - \bar{X})}{(N-1)} 
\]
- Population Covariance between Study and Auxiliary Variables

\[
C_y = \frac{S_y}{\bar{Y}} 
\]
- Population Coefficient of Variation of Study Variable Y

\[
C_x = \frac{S_x}{\bar{X}} 
\]
- Population Coefficient of Variation of Auxiliary Variable X

\[
u_r = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})^r 
\]
- Population rth moment about the mean of Auxiliary Variable X

\[
\beta_3(x) = \frac{u_3}{u_2^2} 
\]
- Population Coefficient of Skewness of Auxiliary Variable X

\[
\beta_2(x) = \frac{u_4}{u_2^2} 
\]
- Population Coefficient of Kurtosis of Auxiliary Variable X

\[
HL = Median[(X_j + X_k)/2, 1 \leq j \leq k \leq N] 
\]
- Hodges-Lehmann Estimator

\[
MR = \frac{X_{(i)} - X_{(N)}}{2} 
\]
- Population mid-range of Auxiliary Variable

\[
G = \frac{4}{N(N-1)} \sum_{i=1}^{N} \left(\frac{2i - N - 1}{2N}\right)X_{(i)} 
\]
- Gini’s Mean Difference for Auxiliary Variable [79]

\[
D = \frac{2\sqrt{\pi}}{N-1} \sum_{i=1}^{N} \left(i - \frac{N+1}{2N}\right)X_{(i)} 
\]
- Downton’s Method for Auxiliary Variable [80]

\[
QD = \frac{Q_3 - Q_1}{2} 
\]
- Population Quartile Deviation of Auxiliary Variable

\[
DM = \frac{D_1 + D_2 + ... + D_9}{9} 
\]
- Decile Mean for Auxiliary Variable

\[
TM = - 
\]
- Trim Mean for Auxiliary Variable
\[
S_{pw} = \frac{\sqrt{\pi}}{N^2} \sum_{i=1}^{N} (2i - N - 1)X_i
\]
- Probability Weighted Moments for Auxiliary Variable

\[R_h = \frac{Y}{X_h}, \quad h = 1, 2, \ldots r\]

\[S_{x_k}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (X_{hi} - \bar{X}_h)^2, \quad h = 1, 2, \ldots r\]

\[S_{y_k} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (X_{hi} - \bar{X}_h)(Y_{hi} - \bar{Y}_h), \quad h = 1, 2, \ldots r\]

\[\varphi_h = \frac{A_h \bar{X}_h}{A_h \bar{X}_h + B_h}, \quad h = 1, 2, \ldots r\]

\[\bar{X}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} X_{hi}, \quad h = 1, 2, \ldots r\]

\[A_j \text{ and } B_j \text{ are any of coefficients of variation, skewness, kurtosis and standard deviation of auxiliary variable } X_j.\]

MSEs = Mean Square Errors
PRE = Percentage Relative Efficiency

2 Literature Review

Several authors have proposed different regression-type estimators using different auxiliary information. The notable ones include Kadilar and Cingi [67], Kadilar and Cingi [68], Subramani and Kumarapandiyam (2006), Abid et al. [69], Subzar et al. [35], Subzar et al. (2018a), Subzar et al. [36] and Subzar et al. [73].

Recently, regression-type estimators have been studied using robust regression slopes and non-conventional robust measures of dispersion.

2.1 Some existing regression estimators with robust regression slopes

Zaman & Bulut [1] proposed ratio-type estimators using Hampel M, Huber MM, LTS, and LAD methods (see Fox. [81], Greenwood et al., [82]) instead of coefficients of slope in ratio estimators. The suggested estimators are as below,

\[\bar{y}_{ZB1} = \frac{\bar{y} + \alpha_{\text{hub}(xy)} (\bar{X} - \bar{x})}{\bar{x}} \bar{X}\]  \hfill (1)

\[\bar{y}_{ZB2} = \frac{\bar{y} + \alpha_{\text{hub}(xy)} (\bar{X} - \bar{x})}{\bar{x} + C_x} (\bar{X} + C_x)\]  \hfill (2)
\[
\bar{y}_{ZB3} = \frac{\bar{y} + \alpha_{\text{rob}(zb)}(\overline{X} - \bar{x})}{\bar{x} + \beta_2(x)} [\overline{X} + \beta_2(x)]
\]

(3)

\[
\bar{y}_{ZB4} = \frac{\bar{y} + \alpha_{\text{rob}(zb)}(\overline{X} - \bar{x})}{\bar{x} \beta_2(x) + C_x} [\overline{X} \beta_2(x) + C_x]
\]

(4)

\[
\bar{y}_{ZB5} = \frac{\bar{y} + \alpha_{\text{rob}(zb)}(\overline{X} - \bar{x})}{\bar{x} C_x + \beta_2(x)} [\overline{X} C_x + \beta_2(x)]
\]

(5)

where \( \alpha_{\text{rob}(zb)} \) are coefficients of slope obtained from Hampel M, Huber MM, LTS, and LAD methods.

\[
MSE(\bar{y}_{ZB}) = \frac{1}{n} \left( R_{KC1}^2 S_x^2 + 2 B_{\text{rob}(zb)} R_{KC1} S_x^2 + B_{\text{rob}(zb)}^2 S_x^2 - 2 R_{KC1} S_y - 2 B_{\text{rob}(zb)} S_y + S_y^2 \right)
\]

(6)

where

\[
R_{KC1} = \frac{\bar{y}}{\overline{X}}, \quad R_{KC2} = \frac{\bar{y}}{(\overline{X} + C_x)}, \quad R_{KC3} = \frac{\bar{y}}{(\overline{X} + \beta_2)}, \quad R_{KC4} = \frac{\bar{y} \beta_2}{(\bar{x} \beta_2 + C_x)}, \quad R_{KC5} = \frac{\bar{y} C_x}{(\bar{x} C_x + \beta_2)}
\]

Zaman [2] adopted transformation techniques on the work of Zaman and Bulut [1] and then proposed a general form of estimators as:

\[
t_z = \mu \frac{\bar{y} + \alpha_{\text{rob}(zb)}(\overline{X} - \bar{x})}{\bar{x}} \overline{X} + (1 - \mu) \frac{\bar{y} + \alpha_{\text{rob}(zb)}(\overline{X} - \bar{x})}{(\bar{x} w_1 + w_2)} (\bar{x} w_1 + w_2)
\]

(7)

where \( \mu \) is a real constant to be determined such that the MSE of \( t_z \) is minimum. \( W_1 \neq 0 \) and \( W_2 \) are either real number or the function of known parameters like \( C_x \) and \( \beta_2(x) \).

\[
MSE(t_z) \geq \frac{1 - f}{n} \left( S_y^2 + \psi_m S_x^2 - 2 \psi_m S_{xy} \right) m = 1, 2, 3, 4
\]

(8)

\[
\psi_m = \mu (\phi_{\text{rob}(zb)} + R) + (1 - \mu) (\phi_{\text{rob}(zb)} + \lambda_{KC(m+1)}) \mu = \frac{B_{\text{reg}} + \phi_{\text{rob}(zb)} + \lambda_{KC(m+1)}}{\lambda_{KC(m+1)} - R}, B_{\text{reg}} = \frac{\rho_{xy} S_y}{S_x}
\]

where

2.2 Some existing regression estimators with non-conventional robust measures of dispersion

Yadav and Zaman [3] suggested the following class of estimators of population mean using some conventional and non-conventional parameters of auxiliary variable along with the information on the size of the sample as,

\[
t_{pi} = \frac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x} + \alpha_j)} (\overline{X} + \omega_j), \quad i = 1, 2, \ldots, 8 \text{ and } j = 1, 2, \ldots, 8
\]

(9)
\[ t_{p2} = \frac{\bar{Y} + b(\bar{X} - \bar{x})}{(\bar{X} + \bar{x})} (\bar{X} + \bar{x}) \]  
\[ i = 9,10,...,16 \quad \text{and} \quad j = 1,2,...,8 \]  
\[ (10) \]

\[ t_{p3} = \frac{\bar{Y} + b(\bar{X} - \bar{x})}{(\bar{X} + \bar{x})} (\bar{X} + \bar{x}) \]  
\[ i = 17,18,...,24 \quad \text{and} \quad j = 1,2,...,8 \]  
\[ (11) \]

\[ \text{MSE}(t_{p}) = \frac{1-f}{n} \left( R_{p,s}^2 S_j^2 - \left( 1 - \rho^2 \right) S_j^2 \right), \quad i = 1,2,...,24 \]  
\[ (12) \]

where,

\[ R_{pi} = \frac{\bar{Y}}{\bar{x} + \bar{x}}, \quad i = 1,2,...,8 \quad \text{and} \quad j = 1,2,...,8 \]  
\[ (13) \]

\[ R_{pi} = \frac{\bar{Y} \rho}{\bar{x} \rho + \bar{x}}, \quad i = 9,10,...,16 \quad \text{and} \quad j = 1,2,...,8 \]  
\[ (14) \]

\[ R_{pi} = \frac{\bar{Y} C_x}{\bar{X} C_x + \bar{x}}, \quad i = 17,18,...,24 \quad \text{and} \quad j = 1,2,...,8 \]  
\[ (15) \]

\[ \sigma_i = (QD + n), \quad \sigma_2 = (DM + n), \quad \sigma_3 = (TM + n), \quad \sigma_4 = (MR + n), \quad \sigma_5 = (HL + n), \]  
\[ \sigma_6 = (G + n), \quad \sigma_7 = (D + n), \quad \sigma_8 = (S_{pw} + n) \]

### 3 Proposed Estimators

The suggested estimators are as (13) and (14)

\[ t_{ik} = \frac{\left[ \bar{y} + \alpha_{\hat{r} \hat{b}(z)} (\bar{x} - \bar{x}) \right]}{(\eta_i \bar{x} + \eta_j)} \]  
\[ \left( \eta_i \bar{x} + \eta_j \right) \]  
\[ (13) \]

\[ t_{2k} = \frac{\left[ \bar{y} + \alpha_{\hat{r} \hat{b}(z)} (\bar{x} - \bar{x}) \right]}{(\eta_i \bar{x} + \eta_j) \bar{x}} \quad i = 1,2,3,4,5,6 \]  
\[ (14) \]

where

\[ \eta_i, \eta_j, \quad i,j = 1,2,3, \quad \eta_i \neq \eta_j \in \{ G \times n, D \times n, S_{pw} \times n \} \]  
\[ k = 1,2,...,6 \]  
\[ (15) \]

### 3.1 Biases and MSEs of the classes of proposed estimator \( t_{ik}, \quad k = 1,2,3,4,5,6 \)

**Theorem 1**: The bias & MSE of the suggested classes of estimators \( t_{ik}, \quad k = 1,2,3,4,5,6 \) to first order of approximation are
\begin{align}
\text{Bias}(t_{ik}) &= \frac{1 - f}{n} \left( \frac{\eta_i}{\eta_i \bar{X} + \eta_j} \left( \alpha_{rbst(zb)} + \frac{\bar{Y} \eta_i}{(\eta_i \bar{X} + \eta_j)} \right) S_X^2 - S_{XY} \right) \\
\text{MSE}(t_{ik}) &= \frac{1 - f}{n} \left( S_Y^2 - 2A_i S_{XY} + A_i^2 S_X^2 \right) \\
A_i &= \alpha_{rbst(zb)} + \frac{\eta_i \bar{Y}}{(\eta_i \bar{X} + \eta_j)}
\end{align}

where,

\textbf{Proof:} Bias and MSE \((t_{ik})\) can be defined up to first order of approximation

\begin{align}
\text{Bias}(t_{ik}) &= 2^{-1} \frac{1 - f}{n} \left( \sum_{i=1}^{q} \sum_{j=1}^{q} \Delta_y \left( \hat{\theta}_i - \theta_i \right) \left( \hat{\theta}_j - \theta_j \right) \right) \\
\text{MSE}(t_{ik}) &= \Delta_1 S_Y^2 + \Delta_2 S_X^2 + 2 \Delta_2 S_{XY}
\end{align}

Where \(q\) is the number of the sample means in the estimator, \(q = 2\), \(\hat{\theta}_1 = \bar{y}, \hat{\theta}_2 = \bar{x}, \theta_1 = \bar{Y}, \theta_2 = \bar{X}\), and

\begin{align}
\Delta_y &= \left. \frac{\partial^2 t_{ik}}{\partial \theta_i \partial \theta_j} \right|_{\bar{y}, \bar{x}} \\
\Delta_1 &= \left. \frac{\partial^2 t_{ik}}{\partial y^2} \right|_{\bar{y}, \bar{x}} = \partial \left( \frac{\eta_i \bar{X} + \eta_j}{(\eta_i \bar{X} + \eta_j)} \right) \left\|_{\bar{y}, \bar{x}} \right. = 0 \\
\Delta_2 &= \left. \frac{\partial^2 t_{ik}}{\partial y \partial x} \right|_{\bar{y}, \bar{x}} = \partial \left( -\frac{\eta_i \bar{X} + \eta_j}{(\eta_i \bar{X} + \eta_j)} \right) \left\|_{\bar{y}, \bar{x}} \right. = \frac{\eta_i}{(\eta_i \bar{X} + \eta_j)} \\
\Delta_2 &= \left. \frac{\partial^2 t_{ik}}{\partial x^2} \right|_{\bar{y}, \bar{x}} = 2 \eta_i \left( \frac{\alpha_{rbst(zb)}}{\eta_i \bar{X} + \eta_j} + \frac{\bar{Y} \eta_i}{(\eta_i \bar{X} + \eta_j)^2} \right)
\end{align}

Substitute (19), (20) and (21) in (18), then (15) is obtained.

\begin{align}
\text{MSE}(t_{ik}) &= \Delta_1 S_Y^2 + \Delta_2 S_X^2 + 2 \Delta_2 S_{XY} \\
&= \Delta_1 S_Y^2 + \Delta_2 S_X^2 + 2 \Delta_2 S_{XY} \quad \text{for} \quad k = 1, 2, 3, 4, 5, 6
\end{align}
where $\Delta$ is a $1 \times 2$ matrix and $\Sigma$ is a $2 \times 2$, variance-covariance matrix defined as:

\[
\Delta = \begin{pmatrix}
\frac{\partial t_{ik}}{\partial y} & \frac{\partial t_{ik}}{\partial x}
\end{pmatrix}_{\tau = -\tau, \tau = \tau}
\]

\[
\Sigma = \begin{pmatrix}
\psi_{n,N} S^2_y & \psi_{n,N} \rho S_y S_X \\
\psi_{n,N} \rho S_y S_X & \psi_{n,N} S^2_X
\end{pmatrix}
\]

From definition, we have,

\[
\frac{\partial t_{ik}}{\partial y} \bigg|_{\tau = -\tau, \tau = \tau} = \frac{\eta_i \bar{X} + \eta_j}{\eta_i \bar{x} + \eta_j} = 1
\]

\[
\frac{\partial t_{ik}}{\partial x} \bigg|_{\tau = -\tau, \tau = \tau} = \left( \frac{\eta_i \bar{X} + \eta_j}{\eta_i \bar{x} + \eta_j} \right) \left( \hat{\alpha}_{rbst(zb)} (\eta_i \bar{x} + \eta_j) + \left( \bar{Y} + \hat{\alpha}_{rbst(zb)} (\bar{X} - \bar{x}) \right) \right)
\]

\[
\frac{\partial t_{ik}}{\partial x} \bigg|_{\tau = -\tau, \tau = \tau} = \left( \hat{\alpha}_{rbst(zb)} + \frac{\bar{Y} \eta_j}{\eta_i \bar{x} + \eta_j} \right) = -A_i \text{ (say)}
\]

\[
\Delta = (1 - A_i)
\]

Substitute (25) in (22), then (16) is obtained, hence, the proof.

### 3.2 Biases and MSEs of classes of the proposed estimator $t_{2k}, k = 1, 2, 3, 4, 5, 6$

**Theorem 2**: The bias and MSE of the suggested estimator $t_{2k}$ ($t_{2k}, k = 1, 2, 3, 4, 5, 6$) to first order of approximation are

\[
\text{Bias}(t_{2k}) = \frac{1 - f}{n} \left( \frac{\hat{\alpha}_{rbst(zb)} + \frac{\bar{Y} \eta_j}{\eta_i \bar{x} + \eta_j}}{\bar{X}^2} + \frac{(1 - \lambda) \left( \hat{\alpha}_{rbst(zb)} (\eta_i \bar{x} + \eta_j) + \bar{Y} \right)}{\eta_i \bar{x} + \eta_j} \right) S_y^2
\]

\[
- \frac{1}{2} \left( \frac{\lambda}{\bar{X}} + \frac{\eta_i (1 - \lambda)}{\eta_i \bar{x} + \eta_j} \right) S_{xy}
\]

\[
MSE(t_{2k}) = \frac{1 - f}{n} \left( S_y^2 - 2A_2 S_{xy} + A_2^2 S_X^2 \right)
\]

\[
A_2 = \lambda \left( \hat{\alpha}_{rbst(zb)} + R \right) + (1 - \lambda) \left( \hat{\alpha}_{rbst(zb)} + \frac{\bar{Y} \eta_j}{(\eta_i \bar{x} + \eta_j)} \right)
\]

\[
\lambda = \frac{\rho_y S_y}{S_X} - \frac{\hat{\alpha}_{rbst(zb)} + \frac{\bar{Y} \eta_j}{(\eta_i \bar{x} + \eta_j)}}{R - \frac{\bar{Y} \eta_j}{(\eta_i \bar{x} + \eta_j)}}
\]

where
**Proof:** $Bias(t_{2k})$ can be defined up to first order of approximation as

$$Bias(t_{2k}) = 2^{-1} \frac{1-f}{n} \left( \sum_{i=1}^{q} \sum_{j=1}^{r} \Delta_{ij} \left( \hat{\theta}_i - \bar{\theta}_i \right) \left( \hat{\theta}_j - \bar{\theta}_j \right) \right)$$

(28)

Where $q$ is the number of the sample means in the estimator, $q = 2$, $\hat{\theta}_i = \bar{y}$, $\hat{\theta}_2 = \bar{x}$, $\theta_1 = \bar{y}$, $\theta_2 = \bar{x}$ and

$$\Delta_{ij} = \frac{\partial^2 t_{2k}}{\partial \hat{\theta}_i \partial \hat{\theta}_j} \bigg|_{\tau = \bar{\tau}, \tau = \bar{x}}$$

Using the above (28), we have,

$$Bias(t_{2k}) = 2^{-1} \frac{1-f}{n} \left[ \Omega_{11} S_{y}^2 + \Omega_{22} S_{x}^2 + 2\Omega_{12} S_{xy} \right]$$

(29)

where,

$$\Omega_{11} = \left. \frac{\partial^2 t_{2k}}{\partial \bar{y}^2} \right|_{\tau = \bar{\tau}, \tau = \bar{x}} = 0$$

(30)

$$\Omega_{22} = \Omega_{22} = \left. \frac{\partial^2 t_{2k}}{\partial \bar{x}^2} \right|_{\tau = \bar{\tau}, \tau = \bar{x}} = \left. \frac{\partial^2 t_{2k}}{\partial \bar{y} \partial \bar{x}} \right|_{\tau = \bar{\tau}, \tau = \bar{x}} = \left( \frac{\lambda \bar{X}}{\bar{x}} + (1-\lambda) \left( \eta_i \bar{X} + \eta_j \right) \right) \left( \eta_i \bar{X} + \eta_j \right)$$

(31)

$$\Omega_{22} = \left. \frac{\partial^2 t_{2k}}{\partial \bar{x}^2} \right|_{\tau = \bar{\tau}, \tau = \bar{x}} = \left. \frac{\partial^2 t_{2k}}{\partial \bar{y} \partial \bar{x}} \right|_{\tau = \bar{\tau}, \tau = \bar{x}} = \left( \frac{\lambda \bar{X}}{\bar{x}} + (1-\lambda) \left( \eta_i \bar{X} + \eta_j \right) \right) \left( \eta_i \bar{X} + \eta_j \right)$$

(32)

Substitute (30), (31) and (32) in (29), (26) is obtained.

$$MSE(t_{2k}) = \Omega \Sigma \Omega^T$$

(33)

where $\Omega$ is a 1x2 matrix and $\Sigma$ is a 2x2 variance-covariance matrix defined as;
\[ \Omega = \left( \frac{\partial t_{2k}}{\partial \eta} \right)_{\eta=\hat{\eta}, \tau=\hat{\tau}} \left( \frac{\partial t_{2k}}{\partial \lambda} \right)_{\eta=\hat{\eta}, \tau=\hat{\tau}} \cdot \Sigma = \left( \frac{\psi_{n,N} S_y^2}{\psi_{n,N} \rho S_y S_x} \right) \cdot \frac{1}{n} - \frac{f}{N} \]

Using the above definition, we obtained

\[ \frac{\partial t_{2k}}{\partial \eta} = \frac{\eta \hat{X} + \eta_j}{\eta \hat{X} + \eta_j} = 1 \]

\[ \frac{\partial t_{2k}}{\partial \lambda} = -\lambda \hat{X} \left( \frac{\hat{\alpha}_{rhat}(\hat{z}) \hat{x} + (\bar{y} + \hat{\alpha}_{rhat}(\hat{z}) \hat{X} - \bar{x})}{\hat{x}^2} \right) \]

\[ = - \left( 1 - \lambda \right) \left( \eta \hat{X} + \eta_j \right) \left( \frac{\hat{\alpha}_{rhat}(\hat{z}) \left( \eta \hat{x} + \eta_j \right) + (\bar{y} + \hat{\alpha}_{rhat}(\hat{z}) \hat{X} - \bar{x}) \eta_j}{\left( \eta \hat{x} + \eta_j \right)^2} \right) \]

\[ \frac{\partial t_{2k}}{\partial \lambda} = - \left( \lambda \left( \hat{\alpha}_{rhat}(\hat{z}) + R \right) + (1 - \lambda) \left( \hat{\alpha}_{rhat}(\hat{z}) + \frac{\bar{y} \eta_j}{\left( \eta \hat{X} + \eta_j \right)} \right) \right) = -A_2 \text{ (say)} \]

\[ \Omega = (1 - A_2) \]  \hspace{2cm} (36)

Substitute (36) in (33), then (27) is obtained.

Differentiate (27) with respect to \( \hat{\lambda} \), equate the result to zero and solve for \( \hat{\lambda} \), the optimum value of \( \hat{\lambda} \) is obtained as

\[ \hat{\lambda} = \frac{\rho S_y S_x}{S_y} \left( \hat{\alpha}_{rhat}(\hat{z}) + \frac{\bar{y} \eta_j}{\left( \eta \hat{X} + \eta_j \right)} \right) \left( R - \frac{\bar{y} \eta_j}{\left( \eta \hat{X} + \eta_j \right)} \right) \]  \hspace{2cm} (37)

4 Empirical Study

In this section, simulation studies were conducted to assess the performance of the proposed estimators with respect to Zaman & Bulut [1] and Zaman [1] estimators. Data of size 1000 units were generated for study populations using function defined in Table 1. Samples of size 100 units were selected randomly 10,000 times by method of simple random sampling without replacements (SRSWOR). The MSEs and PREs of the considered estimators were computed using (38) and (39) respectively.
Table 1. Populations Used for Empirical Study on Zaman & Bulut [1], Zaman [2] and Proposed Estimators $t_{1k}$ and $t_{2k}$

<table>
<thead>
<tr>
<th>Populations</th>
<th>Auxiliary variable (x)</th>
<th>Methods for Robust regression slope $\alpha_{\text{robust}}$</th>
<th>Study variable (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$X \sim \text{pois}(0.1)$</td>
<td>Huber MM, Hampel M, Least Trimmed Squares (LTS), Least Absolute Deviation (LAD)</td>
<td>$Y = 20X^2 + e$, where $e \sim (0, 4)$</td>
</tr>
<tr>
<td>II</td>
<td>$X \sim \text{exp}(0.7)$</td>
<td>Huber MM, Hampel M, Least Trimmed Squares (LTS), Least Absolute Deviation (LAD)</td>
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</tr>
</tbody>
</table>

Table 2. MSEs and PREs of the proposed and some existing estimators using population I under huber MM method

<table>
<thead>
<tr>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Mean $\bar{Y}$</td>
<td>4.156443</td>
<td>100</td>
<td>$t_{p5}$</td>
<td>0.3708674</td>
<td>1120.736</td>
<td>$t_{p22}$</td>
<td>0.3694504</td>
<td>1125.034</td>
</tr>
<tr>
<td>$\bar{Y}$</td>
<td>6.432489</td>
<td>64.6164</td>
<td>$t_{p6}$</td>
<td>0.3702848</td>
<td>1122.499</td>
<td>$t_{p23}$</td>
<td>0.3693983</td>
<td>1125.193</td>
</tr>
<tr>
<td>$\bar{Y}_{ZB1}$</td>
<td>0.3794472</td>
<td>1095.394</td>
<td>$t_{p7}$</td>
<td>0.3702075</td>
<td>1122.733</td>
<td>$t_{p24}$</td>
<td>0.369398</td>
<td>1125.194</td>
</tr>
<tr>
<td>$\bar{Y}_{ZB2}$</td>
<td>0.3905998</td>
<td>1064.118</td>
<td>$t_{p8}$</td>
<td>0.3702069</td>
<td>1122.735</td>
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<tr>
<td>$\bar{Y}_{ZB3}$</td>
<td>0.4454944</td>
<td>932.9955</td>
<td>$t_{p9}$</td>
<td>7.476133</td>
<td>55.59616</td>
<td>$t_{p11}$</td>
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<tr>
<td>$\bar{Y}_{ZB4}$</td>
<td>0.3804599</td>
<td>1092.479</td>
<td>$t_{p10}$</td>
<td>7.476133</td>
<td>55.59616</td>
<td>$t_{p12}$</td>
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<tr>
<td>$\bar{Y}_{ZB5}$</td>
<td></td>
<td></td>
<td>$t_{p11}$</td>
<td>7.476133</td>
<td>55.59616</td>
<td>$t_{p13}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{Y}$</td>
<td></td>
<td></td>
<td></td>
<td>0.3708734</td>
<td>1120.718</td>
<td>$t_{p14}$</td>
<td>0.3644196</td>
<td>1140.565</td>
</tr>
<tr>
<td>$t_{ZM1}$</td>
<td>3084.196</td>
<td>0.1347658</td>
<td>$t_{p12}$</td>
<td>0.3708734</td>
<td>1120.718</td>
<td>$t_{14}$</td>
<td>0.3644196</td>
<td>1140.565</td>
</tr>
<tr>
<td>$t_{ZM2}$</td>
<td>3169.226</td>
<td>0.1311501</td>
<td>$t_{p13}$</td>
<td>0.3703114</td>
<td>1122.418</td>
<td>$t_{15}$</td>
<td>0.3652373</td>
<td>1138.012</td>
</tr>
<tr>
<td>$t_{ZM3}$</td>
<td>2239.978</td>
<td>0.1855573</td>
<td>$t_{p14}$</td>
<td>0.3702362</td>
<td>1122.646</td>
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<td>$t_{ZM4}$</td>
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## Table 3. MSEs and PREs of the proposed and some existing estimators using population I under hampel M method

<table>
<thead>
<tr>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Mean ( \bar{y} )</td>
<td>4.156443</td>
<td>100</td>
<td>( t_{p6} )</td>
<td>0.3702848</td>
<td>112.499</td>
<td>( t_{p23} )</td>
<td>0.3693983</td>
<td>112.913</td>
</tr>
<tr>
<td>( \bar{y}_{ZB1} )</td>
<td>6.372284</td>
<td>65.2269</td>
<td>( t_{p7} )</td>
<td>0.3702075</td>
<td>112.733</td>
<td>( t_{p24} )</td>
<td>0.369398</td>
<td>112.914</td>
</tr>
<tr>
<td>( \bar{y}_{ZB2} )</td>
<td>0.3813039</td>
<td>1090.061</td>
<td>( t_{p8} )</td>
<td>0.3702069</td>
<td>112.735</td>
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<tr>
<td>( \bar{y}_{ZB3} )</td>
<td>0.3931105</td>
<td>1057.322</td>
<td>( t_{p9} )</td>
<td>0.3702133</td>
<td>112.733</td>
<td>( t_{p24} )</td>
<td>0.3693983</td>
<td>112.914</td>
</tr>
<tr>
<td>( \bar{y}_{ZB4} )</td>
<td>0.4404676</td>
<td>943.6433</td>
<td>( t_{p10} )</td>
<td>0.3702075</td>
<td>112.733</td>
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<tr>
<td>( \bar{y}_{ZB5} )</td>
<td>0.3823841</td>
<td>1086.981</td>
<td>( t_{p11} )</td>
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</table>

**Proposed Estimator** \( t_1 \)

<table>
<thead>
<tr>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ZM_1 )</td>
<td>3073.428</td>
<td>0.135238</td>
<td>( t_{p12} )</td>
<td>0.3708734</td>
<td>1120.718</td>
<td>( t_{14} )</td>
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<tr>
<td>( ZM_2 )</td>
<td>3158.318</td>
<td>0.1316031</td>
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<td>0.3708734</td>
<td>1120.718</td>
<td>( t_{15} )</td>
<td>0.3655662</td>
<td>1136.988</td>
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<tr>
<td>( ZM_3 )</td>
<td>2231.113</td>
<td>0.1862946</td>
<td>( t_{p14} )</td>
<td>0.3703114</td>
<td>1122.418</td>
<td>( t_{16} )</td>
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</tr>
<tr>
<td>( ZM_4 )</td>
<td>3082.177</td>
<td>0.1348541</td>
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<td>0.3702362</td>
<td>1122.646</td>
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</table>

**Proposed Estimator** \( t_2 \)
<table>
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<tr>
<th>Estimators</th>
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<th>PREs</th>
<th>Estimators</th>
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<th>PREs</th>
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<th>PREs</th>
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<tr>
<td>Yadav and Zaman</td>
<td>7.476133</td>
<td>55.59616</td>
<td>Yadav and Zaman</td>
<td>7.476133</td>
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<td>Yadav and Zaman</td>
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<tr>
<td>( t_p^1 )</td>
<td>7.476133</td>
<td>55.59616</td>
<td>( t_p^1 )</td>
<td>7.476133</td>
<td>55.59616</td>
<td>( t_p^1 )</td>
<td>7.476133</td>
<td>55.59616</td>
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<tr>
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<td>7.476133</td>
<td>55.59616</td>
<td>( t_p^2 )</td>
<td>7.476133</td>
<td>55.59616</td>
<td>( t_p^2 )</td>
<td>7.476133</td>
<td>55.59616</td>
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</tr>
<tr>
<td>( t_p^3 )</td>
<td>0.3708674</td>
<td>1120.736</td>
<td>( t_p^3 )</td>
<td>0.3708553</td>
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<td>( t_p^3 )</td>
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<td>( t_p^4 )</td>
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<td>( t_p^4 )</td>
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<td>( t_p^4 )</td>
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</table>

Table 4. MSEs and PREs of the proposed and some existing estimators using population i under LTS method

<table>
<thead>
<tr>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
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<th>MSEs</th>
<th>PREs</th>
<th>Estimators</th>
<th>MSEs</th>
<th>PREs</th>
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</thead>
<tbody>
<tr>
<td>Sample Mean ( \bar{y} )</td>
<td>4.222585</td>
<td>100</td>
<td>( t_p^5 )</td>
<td>0.3566033</td>
<td>1184.113</td>
<td>( t_p^{22} )</td>
<td>0.3549082</td>
<td>1189.768</td>
<td>( t_p^{11} )</td>
<td>0.3498859</td>
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<tr>
<td>Zaman &amp; Bulut [1]</td>
<td>6.824481</td>
<td>61.87408</td>
<td>( t_p^6 )</td>
<td>0.3559334</td>
<td>1186.341</td>
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<td>( \bar{y}_{ZB1} )</td>
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<tr>
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Table 5. MSEs and PREs of the proposed and some existing estimators using population I under LAD method

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## Table 6. MSEs and PREs of the proposed and some existing estimators using population II Huber MM method

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<th>PREs</th>
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Table 7. MSEs and PREs of the proposed and some existing estimators using population II hampel M method

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Table 8. MSEs and PREs of the proposed and some existing estimators using population II LTS method

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Table 9. MSEs and PREs of the proposed and some existing estimators using population II LAD method

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\[
MSE(T) = \frac{1}{10000} \sum_{d=1}^{10000} (T - \bar{Y})^2
\]

(38)

\[
PRE(T) = \left( \frac{MSE(\bar{Y})}{MSE(T)} \right) \times 100
\]

(39)

where \( T \) is any of the proposed and existing estimators.

Tables 2, 3, 4 and 5 show the results of MSEs and PREs of the proposed estimators and that of the existing estimators using data from population I under Huber MM, Hampel M, LTS and LAD methods respectively. The result revealed that the proposed estimators have minimum MSEs and higher PREs compared to conventional and other related estimators considered in this study. This implies that the proposed estimators are more efficient and can produce better estimate of the population parameters than the existing estimators.

Table 6 shows the results of MSEs and PREs of the proposed estimators and that of the existing estimators using data from population II under Huber MM method. The result revealed that the proposed estimators have minimum MSEs and higher PREs compared to other existing estimators with the exception of some members of proposed estimators \( t_{11}, t_{12}, t_{14}, \) and \( t_{16} \) which performed below Zaman [2] and Yadav & Zaman [3]. This implies that the proposed estimators \( t_{13}, t_{15}, t_{21}, t_{22}, t_{23}, t_{24}, t_{25}, \) and \( t_{26} \) are more efficient and can produce better estimate of the population parameters than the existing estimators.

Table 7 shows the results of MSEs and PREs of the proposed estimators and that of the existing estimators using data from population II under Hampel M method. The result revealed that the proposed estimators have minimum MSEs and higher PREs compared to other existing estimators with the exception of some members of proposed estimators \( t_{13} \) and \( t_{15} \) which performed below Zaman & Bulut [1], Zaman [2] and Yadav & Zaman [3]. This implies that the proposed estimators \( t_{11}, t_{12}, t_{14}, t_{16}, t_{21}, t_{22}, t_{23}, t_{24}, t_{25}, \) and \( t_{26} \) are more efficient and can produce better estimate of the population parameters than the existing estimators.

Table 8 shows the results of MSEs and PREs of the proposed and other estimators under population II using Least Trimmed squares (LTS) method. The proposed classes of estimator \( t_{2k}, (k = 1, 2, 3, 4, 5, 6) \) outperformed all the existing estimators considered in the study. The results also revealed that the proposed classes of estimator \( t_{1k}, (k = 1, 2, 3, 4, 5, 6) \) are more efficient than the existing sample mean, Zaman & Bulut [1] estimators \( \bar{Y}, \bar{Y}_{ZB2}, \bar{Y}_{ZB3} \) and \( \bar{Y}_{ZB5} \), Zaman [2] estimators with the exception of Zaman & Bulut [1] \( \bar{Y}_{ZB1}, \bar{Y}_{ZB4} \) and \( \bar{Y}_{ZB6} \), Yadav & Zaman [3] estimators.

Table 9 shows the results of MSEs and PREs of the proposed and other estimators under population II using Least Absolute Deviation (LAD) method. The proposed classes of estimator \( t_{2k}, (k = 1, 2, 3, 4, 5, 6) \) outperformed all the existing estimators considered in the study. The results also revealed that the proposed classes of estimator \( t_{1k}, (k = 1, 2, 3, 4, 5, 6) \) are more efficient than the existing sample mean, Zaman & Bulut [1] estimators \( \bar{Y}, \bar{Y}_{ZB2}, \bar{Y}_{ZB3}, \bar{Y}_{ZB5}, \) and \( \bar{Y}_{ZB6} \), Zaman [2] estimators with the exception of Zaman & Bulut [1] \( \bar{Y}_{ZB1}, \bar{Y}_{ZB2}, \bar{Y}_{ZB3}, \bar{Y}_{ZB5}, \) and \( \bar{Y}_{ZB6} \), Yadav & Zaman [3] estimators.
5 Conclusion

From the empirical results, it was revealed that the suggested estimators have minimum MSE compared to other competing estimators considered in the study. Hence, the suggested estimators demonstrated high level of efficiency over the other estimators considered in the study. In the other words, the suggested estimators have higher chance of producing estimate that is closer to the true value of the population mean than other estimators considered in the literature of this study. The suggested estimators are recommended for use in the estimation of population means of any variable of interest especially when the study and auxiliary variables are highly associated or correlated.

Competing Interests

Authors have declared that no competing interests exist.

References


[34] Tailor R, Chouhan S, Garg N. A ratio-cum-product estimator of population mean in stratified random sampling using two auxiliary variables. STATISTICA, anno LXXII, n.3; 2012.


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