Estimation of Finite Population Mean Using an Improved Class of Mixed Estimators with Two Auxiliary Variables

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This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract
This paper deals with a class of estimators of finite population mean using a combination of two mixed classes of estimators by exploring the information on two auxiliary variables. We have assumed that the study variable $y$ is highly correlated with both the auxiliary variables $x$ and $z$. The optimum properties of the proposed class of estimators is studied both theoretically and empirically. The minimum variance bound (MVB) estimator of this class is also derived and compared with several other competing estimators in terms of its bias and percent relative efficiency.

Keywords: Population mean; class of estimators; ratio estimator; dual to product estimator; relative bias; minimum variance bound (MVB); percent relative efficiency (PRE).

1 Introduction
Supplementary information is engaged in various ways in order to design more precise estimators for a finite population of size $N$. The supplementary information can be obtained readily or can be made available by utilizing a very minimum cost from the total cost of survey in order to

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achieve considerable gain in precision of estimators. Sometimes, the information on more than
one auxiliary variable is available which helps the researcher a lot in designing precise estimators
in the form of mixing of different estimators. These mixed estimators can be applied in more
general conditions than the individual ones in terms of correlation structure between the study
variable \((y)\) and auxiliary variables \((x, z)\). For example; to estimate the average cotton output, the
proportion of good seeds and the planting area are two principal auxiliary variables in agricultural
engineering and both of these variables can be utilized to estimate the average cotton output
more precisely. Hence, the auxiliary information is frequently used in the field of medical sciences,
education, biostatistics, biology, economics, management, sociology and many more. Estimation of
finite population parameters, specifically population mean, gained much interest among the survey
samplers. Some of the recent studies can be seen with \([1]\), \([2]\), \([3]\), \([4]\), \([5]\), \([6]\), \([7]\), etc. In the
following, we have noted down some important mixed-type estimators using two auxiliary variables
for estimating finite population mean

\[
Y = N^{-1} \sum_{i=1}^{N} Y_i.
\]

When in information on only as single auxiliary variable is available, \([8]\) proposed ratio estimator
on the basis of a sample of size \(n\) drawn by SRSWOR scheme for estimating \(Y\) as

\[
t_{1_x} = \frac{\bar{y}X}{\bar{y}}, \text{ where } \bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i, \quad (1.1)
\]

and \(\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i\) is the sample mean of study variable \(y\); whose bias and MSE are respectively given
by

\[
B(t_{1_x}) = \bar{Y} \theta (C_{xx} - C_{yx})
\]

\[
MSE(t_{1_x}) = \bar{Y}^2 \theta (C_{yy} + C_{xx} - 2C_{yx}). \quad (1.2)
\]

But, in presence of information on two auxiliary variables, \([9]\) proposed a class of estimators of \(Y\)
by mixing two ratio estimators as

\[
t_1 = [wt_{1_x} + (1 - w)t_{1_z}], \quad (1.3)
\]

where, \(w\) is a positive constant and \(t_{1_x} = \frac{\bar{y}X}{\bar{y}}\) and \(t_{1_z} = \frac{\bar{y}Z}{\bar{y}}\) are two usual ratio estimators of
population mean \(\bar{Y}\) using two auxiliary variables \(x\) and \(z\) respectively at the estimation stage where
\(\bar{Z} = \frac{1}{n} \sum_{i=1}^{n} Z_i\). The bias and mean square error of this class of estimators are

\[
B(t_1) = \bar{Y} \theta [(C_{xx} - C_{yx}) + w(C_{xx} - C_{xz} + C_{xy} - C_{yz})]
\]

\[
MSE(t_1) = \bar{Y}^2 \theta [C_{yy} + C_{xx} - 2C_{yx} + w^2(C_{xx} + C_{xz} - C_{xz})
+2w(C_{yx} - C_{xz} - C_{yz} + C_{xx})]. \quad (1.5)
\]

The value of \(w\) which results a MVB estimator of this class is

\[
w^{(o)} = \frac{(C_{yz} - C_{xz} - C_{yx} + C_{xx})}{C_{xx} + C_{zz} - 2C_{xz}}
\]

and the MVB of this class is

\[
MSE \left(t_1^{(o)}\right) = \bar{Y}^2 \theta \left[C_{yy} + C_{xx} - 2C_{yx} - \frac{(C_{yz} - C_{xz} - C_{yx} + C_{xx})^2}{C_{xx} + C_{zz} - 2C_{xz}}\right]. \quad (1.6)
\]
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10 proposed three such estimators as

\[ t_2 = \bar{y} \left( \frac{\bar{x}}{\bar{z}} \right) \left( \frac{\bar{X}}{\bar{Z}} \right) \]

\[ t_3 = \bar{y} \left( \frac{\bar{x}}{\bar{z}} \right) \left( \frac{\bar{X}}{\bar{Z}} \right) \]

\[ t_4 = \bar{y} \left( \frac{\bar{x}}{\bar{z}} \right) \left( \frac{\bar{X}}{\bar{Z}} \right) \] (1.7)

The bias and MSE of these estimators are

\[ B(t_2) = \bar{y} \theta \left[ C_{xx} + C_{yz} - C_{xz} - C_{yx} \right] \] (1.8)

\[ B(t_3) = \bar{y} \theta \left[ C_{xx} + C_{xz} - C_{yx} - C_{yz} \right] \] (1.9)

\[ B(t_4) = \bar{y} \theta \left[ C_{yx} + C_{yz} - C_{xz} \right] \] (1.10)

\[ MSE(t_2) = \bar{y}^2 \theta \left[ C_{yy} + C_{xx} - 2C_{yx} - 2C_{xz} \right] \] (1.11)

\[ MSE(t_3) = \bar{y}^2 \theta \left[ C_{yy} + C_{xx} - 2C_{yx} + 2C_{xz} \right] \] (1.12)

\[ MSE(t_4) = \bar{y}^2 \theta \left[ C_{yy} + C_{xx} + 2C_{yx} + 2C_{xz} \right] \] (1.13)

11 proposed dual to ratio estimator basing on the auxiliary variable \( x \), which is negatively correlated with \( y \) as

\[ t_5 = \frac{\bar{x}^*}{\bar{X}}, \quad \text{where} \quad \bar{x}^* = (1 + g)\bar{X} - g\bar{y}, \quad g = \frac{n}{N - n} \] (1.14)

The bias and mean square error of this estimator are given by

\[ B(t_5) = -\bar{y} \theta g C_{yx}, \quad \theta = \frac{1}{n} - \frac{1}{N} \] (1.15)

\[ MSE(t_5) = \bar{y}^2 \theta \left[ C_{yy} + g^2 C_{xx} - 2g C_{yx} \right] \] (1.16)

In the similar manner, [12] suggested dual to product estimator for estimating \( \overline{Y} \), when \( y \) and \( x \) are positively correlated, as

\[ t_6 = \frac{\bar{X}}{\bar{X}^2} \] (1.17)

The bias and mean square error of this estimator are given by

\[ B(t_6) = \bar{y} \theta g \left[ C_{xy} + g C_{xx} \right] \] (1.18)

\[ MSE(t_6) = \bar{y}^2 \theta \left[ C_{yy} + g^2 C_{xx} + 2g C_{yx} \right]. \] (1.19)

13 proposed a class of estimators of population mean when the population means \( \bar{X} \) and \( \bar{Z} \) of the auxiliary variables \( x \) and \( z \) are known in advance, which is given by

\[ t_7 = \bar{y} \left( \frac{\bar{x}^1}{\bar{X}} \right)^{\alpha_1} \left( \frac{\bar{z}^2}{\bar{Z}} \right)^{\alpha_2} . \] (1.20)

where \( \alpha_1, \alpha_2 \) are two unknown constant. The bias and mean square error of \( t_7 \) becomes

\[ B(t_7) = \bar{y} \theta \left[ C_{yx} + \alpha_2 C_{yz} + \alpha_1 \alpha_2 C_{xz} \right. \]

\[ + \frac{\alpha_1 (\alpha_1 - 1)}{2} C_{xx} + \frac{\alpha_2 (\alpha_2 - 1)}{2} C_{zz} \] (1.21)

\[ MSE(t_7) = \bar{y}^2 \theta \left[ C_{yy} + \alpha_2^2 C_{xx} + \alpha_1^2 C_{xz} \right. \]

\[ + 2 \alpha_1 C_{yx} + 2 \alpha_2 C_{yz} + 2 \alpha_1 \alpha_2 C_{xz} \] (1.22)
respectively and the values of $\alpha_1$ and $\alpha_2$ which minimizes the $MSE(t_2)$ are
\[
\alpha_1 = -\frac{(\rho_{yx} - \rho_{zx} \rho_{yz})}{(1 - \rho_{z}^2)} \sqrt{C_{yy} \over C_{xx}}, \quad \text{and} \quad \alpha_2 = -\frac{(\rho_{yz} - \rho_{zx} \rho_{xz})}{(1 - \rho_{z}^2)} \sqrt{C_{yy} \over C_{xx}} \tag{1.23}
\]
respectively, where $\rho_{yx}, \rho_{yz}, \rho_{zx}$ are the simple coefficient of Correlation between the variables $y, x$ and $z$, $\rho_{y,z}$ is the multiple correlation coefficient between $y$ on $x$ and $z$. The MVB of this class is given by
\[
MSE(t_2) = \sum \theta C_{yy} (1 - \rho_{y,z}^2) . \tag{1.24}
\]
It can be easily verified that the estimators $t_2, t_3$ and $t_4$ proposed by [10] are the particular members of this class $t_2$.

[14] proposed bivariate dual to product estimator for estimating $\tau$ using two auxiliary variables as
\[
t_8 = \frac{X}{Y} \frac{Z}{Z'}, \quad \text{where} \quad \tau' = (1 + g)Z - gZ. \tag{1.25}
\]
whose bias and mean square error are given by
\[
B(t_8) = \sum \theta \left[ g^2 (C_{xx} + C_{xz} + C_{xz}) + g(C_{yx} + C_{yx}) \right] \tag{1.26}
\]
\[
MSE(t_8) = \sum \theta^2 \left[ C_{yy} + g^2 (C_{xx} + C_{xz} + 2C_{xz}) + 2g(C_{yx} + C_{yx}) \right] \tag{1.27}
\]
respectively. [15] proposed dual to ratio cum product estimator for estimating $\tau$ as
\[
t_9 = \frac{\tau'}{A} \frac{Z}{Z'}, \tag{1.28}
\]
whose bias and mean square error are given by
\[
B(t_9) = \sum \theta \left[ gC_{yx} + g^2 C_{xx} - gC_{yx} - g^2 C_{xx} \right] \tag{1.29}
\]
\[
MSE(t_9) = \sum \theta^2 \left[ C_{yy} + g^2 (C_{xx} + C_{xx} - 2C_{xx}) + 2gC_{yx} - 2gC_{yx} \right] \tag{1.30}
\]
respectively. [16] proposed a general family of dual to ratio cum product estimators for estimating population mean $\bar{Y}$ following [17] as
\[
t_{10} = \bar{Y} \left( \frac{\tau'}{A} \right)^{\psi_1} \left( \frac{Z'}{Z'} \right)^{\psi_2} \tag{1.31}
\]
where $\psi_1, \psi_2$ are two suitably chosen constants. The bias and MSE of this class $t_{10}$ are
\[
B(t_{10}) = \sum \theta \left[ \psi_2 gC_{yx} - \psi_1 gC_{yx} - \psi_1 g^2 C_{xx} + \frac{\psi_1 (\psi_1 - 1)}{2} \frac{g^2 C_{xx}}{2} \frac{g^2 C_{xx}}{2} \right], \tag{1.32}
\]
\[
MSE(t_{10}) = \sum \theta^2 \left[ C_{yy} + g^2 \left( \psi_1^2 C_{xx} + \psi_1^2 C_{xx} - 2\psi_1 \psi_2 C_{xx} \right) - 2g(\psi_1 C_{yx} - \psi_2 C_{yx}) \right]. \tag{1.33}
\]
The optimum values of $\psi_1$ and $\psi_2$ which minimizes $MSE(t_{10})$ are
\[
\psi_1 = \frac{(\rho_{yx} - \rho_{zx} \rho_{yz})}{g(1 - \rho_{z}^2)} \sqrt{C_{yy} \over C_{xx}}, \quad \psi_2 = \frac{(\rho_{yz} - \rho_{zx} \rho_{xz})}{g(1 - \rho_{z}^2)} \sqrt{C_{yy} \over C_{xx}} \tag{1.34}
\]
The MVB of this class is
\[
MSE(t_{10}) = \sum \theta^2 C_{yy} \theta \left[ 1 - \rho_{y,z}^2 \right]. \tag{1.35}
\]
It can be easily verified that the estimators proposed by [14] and [15] are particular members of this class \( t_{10} \). [18] proposed a class of ratio cum dual to product estimator for estimating \( \bar{y} \) as

\[
t_{11} = \frac{y}{\overline{x}} \left[ \frac{\alpha}{\overline{x}} + (1 - \alpha) \frac{\overline{x}}{\overline{y}} \right]
\]  

(1.36)

where, \( \alpha \) is a real constant. The bias and mean square error of \( t_{11} \) becomes

\[
B(t_{11}) = \overline{y} \theta \left[ g(C_{yx} + gC_{xx}) + \alpha(1 + g) \{(1 - g)C_{xx} - C_{yx}\} \right] 
\]  

(1.37)

\[
MSE(t_{11}) = \overline{y}^2 \theta \left[ C_{yy} + \{g - \alpha(1 + g)\}^2 C_{xx} + 2\{g - \alpha(1 + g)\} C_{yx} \right] 
\]  

(1.38)

and the optimum value of \( \alpha \) which results an MVB for this class is given by

\[
\alpha^{(o)} = \frac{C_{yx} + gC_{xx}}{(1 + g)C_{xx}}
\]

and the MVB of this class is given by

\[
MSE(t_{11}^{(o)}) = \overline{y}^2 \theta C_{yy} \left[ 1 - \rho_{yy}^2 \right].
\]  

(1.39)

Again, [19] suggested dual to product cum dual to ratio estimator as

\[
t_{12} = \frac{y}{\overline{x}} \left[ \frac{\alpha}{\overline{x}} + (1 - \alpha) \frac{\overline{x}}{\overline{y}} \right],
\]  

(1.40)

where \( \alpha \) is any scalar. The bias and mean square error of the estimator \( t_{12} \) is given by

\[
B(t_{12}) = \overline{y} \theta g \left[ 2\alpha - 1 \right] C_{yx} + \alpha g C_{xx}
\]  

(1.41)

\[
MSE(t_{12}) = \overline{y}^2 \theta \left[ C_{yy} + g(2\alpha - 1) \left\{ g(2\alpha - 1) C_{xx} + 2C_{yx} \right\} \right]
\]  

(1.42)

The optimum value of \( \alpha \) which results an MVB for this class is given by

\[
\alpha^{(o)} = \frac{-C_{yx} - gC_{xx}}{2gC_{XX}}
\]

and the MVB of this class is the MVB of the class \( t_{11} \).

2 The Proposed Class of Estimators

Motivated by the above estimators, we propose a class of estimators \( t \) for the finite population mean \( \bar{y} \) by combining the ratio estimator along with dual to product estimator as

\[
t = \frac{y}{\overline{x}} \left[ \alpha \left( \frac{\overline{x}}{\overline{y}} \right) + (1 - \alpha) \left( \frac{\overline{y}}{\overline{x}} \right) \right] \left[ \beta \left( \frac{\overline{y}}{\overline{x}} \right) + (1 - \beta) \left( \frac{\overline{x}}{\overline{y}} \right) \right],
\]  

(2.1)

where \( \alpha \) and \( \beta \) are two real constants or parameters. In order to study the large sample behaviour of this estimator, we consider

\[
\overline{y} = \overline{y}(1 + e_i), \quad \overline{x} = \overline{x}(1 + e_i), \quad \overline{z} = \overline{z}(1 + e_i), \quad E(e_i) = 0, \quad i = 0, 1, 2.
\]  

(2.2)

So, \( e_i \)'s are the sampling errors associated with respective statistics. Thus, we have

\[
E(e_i^0) = \theta C_{yy}, \quad E(e_i^1) = \theta C_{xx}, \quad E(e_i^2) = \theta C_{zz}
\]

\[
E(e_0 e_1) = \theta C_{yx}, \quad E(e_0 e_2) = \theta C_{yz}, \quad E(e_1 e_2) = \theta C_{xz}.
\]  

(2.3)

Using (2.2) and (2.3) in (2.1) we get

\[
t = \overline{y} \left[ 1 + e_0 + g e_1 + g e_2 + g e_0 e_1 + g e_0 e_2 + g^2 e_1^2 + g^2 e_2^2 + g^2 e_1 e_2 + \alpha (e_1^0 - g e_1^2 - e_1 - e_0 e_1 - g e_0 e_1 - e_1 - e_0 e_1) + \alpha \beta (g e_1 e_2 + g^2 e_1 e_2 + e_1 e_2 + g e_1 e_2) \right] + o(e_i^2).
\]  

(2.4)
3 Properties of the Proposed Class

The equation (2.4) immediately implies the bias of the proposed class $t$ as

$$B(t) = T\theta \left[g^2(C_{xx} + C_{xz} + C_{xz}) + g(C_{yx} + C_{yy}) + \beta(1 + g)\{(1 - g)C_{xz} - C_{yz} - gC_{xx}\} + \alpha(1 + g)\{(1 - g)C_{xx} - C_{yz} - gC_{xz}\} + \alpha\beta(1 + g)\{(1 + g)C_{xz}\}\right]$$

and MSE up to $o(n^{-1})$ as

$$MSE(t) = T\theta^2 \left[C_{yy} + g^2(C_{xx} + C_{xz} + 2C_{xz}) + (1 + g)^2(\beta^2C_{xx} + \alpha^2C_{xz} + 2\alpha\betaC_{xz}) + 2g(C_{yx} + C_{yz}) - 2g(1 + g)(\beta C_{xz} + \alpha C_{xx} + \beta C_{xx} + \alpha C_{xz}) - 2(1 + g)(\beta C_{yy} + \alpha C_{yz})\right]$$

The optimum values of $\alpha$ and $\beta$ which minimizes the $MSE(t)$ are given by

$$\alpha^{(o)} = \frac{1 + g}{1 + g} + \frac{(\rho_{yz} - \rho_{xz})}{(1 + g)(1 - \rho_{z}^2)} \sqrt{\frac{C_{yy}}{C_{zz}}}$$

$$\beta^{(o)} = \frac{1 + g}{1 + g} + \frac{(\rho_{yz} - \rho_{xz})}{(1 + g)(1 - \rho_{z}^2)} \sqrt{\frac{C_{yy}}{C_{zz}}}$$

Using the optimum values of $\alpha^{(o)}$ and $\beta^{(o)}$ in place of $\alpha$ and $\beta$ in equation (3.2), we get the MVB of the class $t$ as

$$MSE(t^{(o)}) = T\theta^2 \left[C_{yy} \theta \left[1 - \rho_{y,xx}^2\right]\right],$$

where $\rho_{y,xx}$ is the multiple correlation coefficient between $y$ on $x$ and $z$ and the corresponding MVB estimator is

$$t^{(o)} = g \left[\frac{X}{\sqrt{\frac{C_{yy}}{C_{xx}}}} \left\{g + \frac{(\rho_{yz} - \rho_{xz})}{(1 - \rho_{z}^2)} \right\} \frac{X}{\sqrt{\frac{C_{yy}}{C_{zz}}}} + \left\{1 - \frac{(\rho_{yz} - \rho_{xz})}{(1 - \rho_{z}^2)} C_{yy} \right\} \frac{X}{\sqrt{\frac{C_{yy}}{C_{zz}}}} \right].$$

4 Some Particular Cases

Some of the popular estimators as the members of proposed class of estimators $t$ are discussed below along with their bias and MSE.

a. Bivariate Ratio Estimator

When $\alpha = \beta = 1$, the proposed class of estimators ‘$t$’ reduces to ratio estimator using two auxiliary variables proposed by [10]

$$t^*_1 = \frac{X}{\sqrt{\frac{C_{yy}}{C_{xx}}}} \frac{Z}{\sqrt{\frac{C_{yy}}{C_{zz}}}}.$$  

The bias of $t^*_1$ is

$$B(t^*_1) = T\theta [C_{xx} + C_{xz} + C_{xz} - C_{yz} - C_{yy}]$$

and the mean squared error is

$$MSE(t^*_1) = T\theta^2 [C_{yy} + C_{xx} + C_{xz} + 2C_{xz} - 2C_{yz} - 2C_{yy}].$$
b. Bivariate Dual to Product Estimator
When $\alpha = \beta = 0$, ‘$t$’ reduces to usual dual to product estimator with two auxiliary variables proposed by [14]

$$t'_2 = \sum_{i} \frac{Y_i Z_i}{x_i z_i}$$  \hspace{1cm} (4.4)$$

Its bias and MSE are given as

$$B(t'_2) = \bar{Y} g [g (C_{xx} + C_{zz} + C_{xz}) + C_{yx} + C_{yz}]$$  \hspace{1cm} (4.5)$$

and

$$MSE(t'_2) = \bar{Y}^2 g^2 \left[ C_{yy} + g^2 (C_{xx} + C_{zz} + 2C_{xz}) + 2(C_{yx} + C_{yz}) \right].$$  \hspace{1cm} (4.6)$$

c. Ratio cum Dual to Product Estimator
When $\alpha = 1, \beta = 0$, the class of estimators ‘$t$’ reduces to the ratio cum dual to product estimator with two auxiliary variables as

$$t'_3 = \sum_{i} \frac{Y_i Z_i}{x_i z_i}.$$  \hspace{1cm} (4.7)$$

The bias and mean square error of this estimators are

$$B(t'_3) = \bar{Y}^2 \theta \left[ C_{xx} + g^2 (C_{xx} + C_{zz}) + g (C_{yx} - C_{xz}) \right] - \left( C_{yx} + C_{xz} \right),$$  \hspace{1cm} (4.8)$$

$$MSE(t'_3) = \bar{Y}^2 \theta^2 \left[ C_{yy} + C_{xx} + g^2 C_{xx} + 2g C_{yx} - 2g C_{xz} - 2g C_{yy} \right].$$  \hspace{1cm} (4.9)$$

respectively. The case when $\alpha = 0, \beta = 1$ is similar, so it is omitted.

5 Comparison with Different Estimators

We compare the mean square error of the proposed class of estimators ‘$t$’ from (3.2) with the different competing estimators proposed by different authors. Again, minimum variance bound (MVB) estimators is a focus among the estimators of any class, so we also compare the MVB of the proposed class of estimators ‘$t$’ with other estimators as well as the MVB of other competing classes.

a. With Mean per Unit Estimator
The variance of mean per unit estimator $t_0 = \bar{y}$ is

$$V(t_0) = \bar{y}^2 \theta C_{yy}$$  \hspace{1cm} (5.1)$$

From (3.5) and (5.1), the proposed class of estimators $t$ is preferred to mean per unit estimator $t_0$ if

$$\rho^2_{y,xz} > 0$$  \hspace{1cm} (5.2)$$

which is always true.

b. With Olkin (1958) Estimator
From (3.5) and (1.5), the proposed class of estimators $t$ is preferred to Olkin (1958) estimator $t_1$ if

$$\frac{(C_{yz} - C_{xz} + C_{yz} + C_{xz})^2}{(C_{xx} + C_{zz} - 2C_{xz})} - C_{zz} + 2C_{yz} - \rho_{y,xz}^2 > 0$$  \hspace{1cm} (5.3)$$

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c. With Abu-Dayyeh et al. (2003) Estimator
From (3.5) and (1.22), the MVB of the proposed class of estimators $t$ and Abu-Dayyeh (2003) Estimator $t_7$ are equal. So, $MSE(t) = MSE(t_7)$

d. With Singh et al. (2011) Estimator
From (3.5) and (1.33), the MVB of the proposed class of estimators $t$ is equal to MVB of Singh (2011) Estimator $t_{10}$. So, $MSE(t) = MSE(t_{10})$.

e. With Choudhury and Singh (2012) Estimator
From (3.5) and (1.42), the proposed class of estimators $t$ is preferred to Choudhury and Singh (2012) Estimator $t_{12}$, if

$$\rho_{yxz}^2 - \rho_{yxt}^2 > 0.$$  \hspace{1cm} (5.4)

6 Empirical Study

In order to study the performance of the proposed class of estimators along with several competing estimators/class of estimators, we list some classes of estimators and their members in Table 1. It is clear that the proposed class of estimators includes a number of popular estimator. Since, we are comparing the precision of various classes of estimators, it is desirable to consider the MVB estimators and their variance/MSE to arrive at a conclusion. So, we consider the comparison of the proposed class $t$ with Olkin (1958) class of estimators $t_1$, Abu-Dayyeh et al. (2003) class $t_7$, Singh et al. (2011) class $t_{10}$ and Choudhury and Singh (2012) class of estimators $t_{12}$, since the other estimator are particular members of these classes.

To examine the behaviour of proposed estimators $t$ we have considered fourteen natural populations which are available in different text books. We consider different characteristics for the comparison between these estimators:

I. Percent Relative Bias of an estimator $T$:

$$PRB(T) = \frac{|E(T) - Y|}{\bar{Y}} \times 100.$$  \hspace{1cm} (6.1)

II. PRE’s of an estimator $T$ with respect to the simple mean estimator $t_0$ is given by

$$PRE(T) = \frac{V(t_0)}{MSE(T)} \times 100.$$  \hspace{1cm} (6.2)

Table 2 gives the sources and description of the variables $(y, x, z)$ and Table 3 describes some selected population parameters. Table 4 gives the values of the constants $(\alpha$ and $\beta$) used in the proposed class of estimators $t$ for which it leads to an MVB estimator and Table 5 gives the bias and Table 6 gives percent relative efficiency (PRE’s) of different classes of estimators (MVB case) with respect to simple mean estimator $t_0$.

From Table 4 we have seen that the bias of proposed estimator is minimum as compared to the other competitive estimators. From Table 5 we have seen that the mean squared error of the proposed estimator is minimum for all the population as compared to the other existing estimators proposed by different authors. From Table 6, the proposed estimator gains maximum percent relative efficiency (PRE) as compared to the other competitive estimators. So, it is found that the proposed estimator is more efficient than the existing estimators.
Table 1. Some Competing Classes of Estimators and Some popular estimators of these Classes

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Classes of Estimators</th>
<th>Some Popular Members</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>[9] Class</td>
<td>( t_1 = \frac{x}{X}, ) [8] ( t_1 = \frac{x}{X}, ) [8]</td>
</tr>
<tr>
<td>2.</td>
<td>Abu-Dayesh(2003) Class</td>
<td>( t_2 = \frac{y}{\left( \frac{X}{Z} \right)} ) ( t_3 = \frac{y}{\left( \frac{X}{Z} \right)}, ) [10] ( t_4 = \frac{y}{\left( \frac{X}{Z} \right)} )</td>
</tr>
<tr>
<td>3.</td>
<td>Singh (2011) Class</td>
<td>( t_5 = \frac{y}{\left( \frac{X}{Z} \right)} ) ( t_6 = \frac{y}{\left( \frac{X}{Z} \right)}, ) [11] ( t_7 = \frac{y}{\left( \frac{X}{Z} \right)} )</td>
</tr>
<tr>
<td>4.</td>
<td>Choudhury (2012) Classes</td>
<td>( t_8 = \frac{y}{\left( \frac{X}{Z} \right)}, ) [14] ( t_9 = \frac{y}{\left( \frac{X}{Z} \right)} )</td>
</tr>
<tr>
<td>5.</td>
<td>Proposed Class</td>
<td>( t_{10} = \frac{y}{\left( \frac{X}{Z} \right)} ) ( t_{11} = \frac{y}{\left( \frac{X}{Z} \right)} ) ( t_{12} = \frac{y}{\left( \frac{X}{Z} \right)} )</td>
</tr>
</tbody>
</table>

Remarks:

I. The percent relative bias of the MVB estimator of the proposed class of estimators is very less indicates that this MVB estimator can be treated as almost unbiased for estimating population mean \( \bar{Y} \).

II. The bias can be reduced by increasing the sample size.

III. The percent relative efficiency (PRE) of the MVB estimator of the proposed class is maximum for all populations.

IV. The MVB of three classes namely proposed class, Abu-Dayyeh(2003) Class \( t_7 \) and Singh (2011) class \( t_{10} \) are equal but the MVB estimators of these classes are different.

V. The PRE of the MVB estimators of the proposed class of estimators \( t \) is same as that of the class of estimators proposed by [16].

VI. But, in comparing the biases of the two MVB estimators of proposed class \( t \) and class of estimators proposed by [16], we could not conclude in favor of any particular class as the minimum bias varies from one population to another.

VI. The proposed class of estimators can be easily extended to multi-auxiliary information case as

\[
T = \frac{P}{\prod_{i=1}^{p} \left( \alpha_i \left( \frac{X_i}{x_i} \right) + (1 - \alpha_i) \left( \frac{X_i}{x_i} \right) \right) \left( \beta_i \left( \frac{Z_i}{z_i} \right) + (1 - \beta_i) \left( \frac{Z_i}{z_i} \right) \right)},
\]

(6.3)

where \( x_1, x_2, \ldots, x_p \) are \( p \)-auxiliary variables and the minimum variance bound for this class of estimators is equal to that of the multivariate linear regression estimator which is equal to

\[
MSE (T^\circ) = \theta^2 C_{yy} \left( 1 - \rho_{23}^2 \right),
\]

(6.4)
where $\rho_{23...p}^2$ is the square of the multiple regression coefficient of $y$ on $x_1, x_2, \cdots, x_p$.

### Table 2. Sources and descriptions

<table>
<thead>
<tr>
<th>Pop. No.</th>
<th>Source</th>
<th>$y$</th>
<th>$x$</th>
<th>$z$</th>
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</thead>
<tbody>
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<td>1</td>
<td>[20], p.203</td>
<td>Actual Inflation Rate</td>
<td>Unemployment rate</td>
<td>Unexpected inflation rate</td>
</tr>
<tr>
<td>2</td>
<td>[20], p.216</td>
<td>Real Gross Product (Millions of NT$)</td>
<td>Labour days (Millions of days)</td>
<td>Real capital input (Millions of NT$)</td>
</tr>
<tr>
<td>3</td>
<td>[20], p.224</td>
<td>Real Gross Product (Millions of NT$)</td>
<td>Labour input (per thousand persons)</td>
<td>Real Capital input (Millions of NT$)</td>
</tr>
<tr>
<td>4</td>
<td>[20], p.227</td>
<td>Defense budget outlay for years $t$ $/$billions</td>
<td>GNP in different years, $/$billions</td>
<td>U.S. military sales/assistance</td>
</tr>
<tr>
<td>5</td>
<td>[20], p.227</td>
<td>Defense budget outlay for years $t$ $/$billions</td>
<td>GNP in different years, $/$billions</td>
<td>Average industry sales</td>
</tr>
<tr>
<td>6</td>
<td>[20], p.228</td>
<td>Per capita consumption of Chicken, Ibs</td>
<td>Real disposable income per capita, $</td>
<td>Real retail price of chicken per Ib</td>
</tr>
<tr>
<td>7</td>
<td>[20], p.228</td>
<td>Per capita consumption of Chicken, Ibs</td>
<td>Real disposable income per capita, $</td>
<td>Real retail price of pork per Ib</td>
</tr>
<tr>
<td>8</td>
<td>[20], p.228</td>
<td>Per capita consumption of Chicken, Ibs</td>
<td>Real disposable income per capita, $</td>
<td>Real retail price of beef per Ib</td>
</tr>
<tr>
<td>9</td>
<td>[20], p.228</td>
<td>Per capita consumption of Chicken, Ibs</td>
<td>Real disposable income per capita, $</td>
<td>Composite real price of chicken substitutes per Ib</td>
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<tr>
<td>10</td>
<td>[21], p.286</td>
<td>Mean yield of rice per plant</td>
<td>Number of tillers</td>
<td>Percentage of sterility</td>
</tr>
<tr>
<td>11</td>
<td>[22], p.399</td>
<td>Area under wheat in 1964 (in acres)</td>
<td>Area under wheat in 1963 (in acres)</td>
<td>Cultivated area in 1961 (in acres)</td>
</tr>
<tr>
<td>12</td>
<td>[22], p.228</td>
<td>Output of the Factory</td>
<td>The number of workers</td>
<td>Fixed capital</td>
</tr>
<tr>
<td>13</td>
<td>[23], p.1115</td>
<td>Season average price per pound during 1996</td>
<td>Season average price per pound during 1995</td>
<td>Season average price per pound during 1994</td>
</tr>
<tr>
<td>14</td>
<td>[24], p.182</td>
<td>Number of 'placebo' children</td>
<td>Number of paralytic polio cases in the 'placebo' group</td>
<td>Number of paralytic polio cases in the 'not inoculated' group</td>
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</table>
Table 3. Some Selected Characteristics of the Populations

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<th>n</th>
<th>g</th>
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<th>X</th>
<th>Z</th>
<th>C_{yy}</th>
<th>C_{xz}</th>
<th>C_{zz}</th>
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<tbody>
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<td>0.300</td>
<td>7.757</td>
<td>6.654</td>
<td>6.686</td>
<td>0.167</td>
<td>0.051</td>
<td>0.152</td>
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<tr>
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<td>15</td>
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<td>0.250</td>
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<td>333.333</td>
<td>287.347</td>
<td>25506.633</td>
<td>0.042</td>
<td>0.003</td>
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<tr>
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<td>0.500</td>
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<td>159919.333</td>
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<td>0.062</td>
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<td>7</td>
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<td>83.860</td>
<td>83.860</td>
<td>1358.155</td>
<td>6.287</td>
<td>0.126</td>
<td>0.299</td>
</tr>
<tr>
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<td>6</td>
<td>0.429</td>
<td>83.860</td>
<td>83.860</td>
<td>1358.155</td>
<td>29.145</td>
<td>0.126</td>
<td>0.299</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
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<td>0.278</td>
<td>39.670</td>
<td>39.670</td>
<td>1035.065</td>
<td>47.996</td>
<td>0.036</td>
<td>0.373</td>
</tr>
<tr>
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<td>23</td>
<td>6</td>
<td>0.353</td>
<td>39.670</td>
<td>39.670</td>
<td>1035.065</td>
<td>90.400</td>
<td>0.036</td>
<td>0.373</td>
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<tr>
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<td>7</td>
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<td>39.670</td>
<td>1035.065</td>
<td>124.448</td>
<td>0.036</td>
<td>0.373</td>
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<tr>
<td>9</td>
<td>23</td>
<td>5</td>
<td>0.278</td>
<td>39.670</td>
<td>39.670</td>
<td>1035.065</td>
<td>107.857</td>
<td>0.036</td>
<td>0.373</td>
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<tr>
<td>10</td>
<td>50</td>
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<td>0.190</td>
<td>12.830</td>
<td>9.040</td>
<td>9.040</td>
<td>18.762</td>
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<tr>
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<td>34</td>
<td>10</td>
<td>0.417</td>
<td>199.441</td>
<td>747.588</td>
<td>208.882</td>
<td>0.584</td>
<td>0.363</td>
<td>0.535</td>
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<tr>
<td>12</td>
<td>80</td>
<td>18</td>
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<td>199.441</td>
<td>747.588</td>
<td>1126.463</td>
<td>0.127</td>
<td>0.911</td>
<td>0.571</td>
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<tr>
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<td>36</td>
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<td>0.500</td>
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<td>1126.463</td>
<td>0.349</td>
<td>0.191</td>
<td>0.062</td>
</tr>
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<td>14</td>
<td>34</td>
<td>8</td>
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<td>2.588</td>
<td>2.912</td>
<td>1.079</td>
<td>0.171</td>
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</table>

Table 4. Optimum Values of $\alpha^{(o)}$ and $\beta^{(o)}$

<table>
<thead>
<tr>
<th>P. No.</th>
<th>$\alpha^{(o)}$</th>
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<td>1</td>
<td>-0.088</td>
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<td>1.481</td>
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<tr>
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<td>1.481</td>
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<tr>
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<td>1.481</td>
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<tr>
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<td>1.481</td>
</tr>
<tr>
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<td>1.306</td>
<td>1.481</td>
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<tr>
<td>7</td>
<td>1.306</td>
<td>1.481</td>
</tr>
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<td>8</td>
<td>1.306</td>
<td>1.481</td>
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<tr>
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<td>1.306</td>
<td>1.481</td>
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<tr>
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<td>1.306</td>
<td>1.481</td>
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<td>1.306</td>
<td>1.481</td>
</tr>
<tr>
<td>12</td>
<td>1.306</td>
<td>1.481</td>
</tr>
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</table>

Table 5. Relative Bias of the MVB Estimator of Different Classes of Estimators

<table>
<thead>
<tr>
<th>P. No.</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_{10}$</th>
<th>$t_{12}$</th>
<th>$t$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.012</td>
<td>0.304</td>
<td>0.069</td>
<td>0.006</td>
<td>0.209</td>
</tr>
<tr>
<td>2</td>
<td>0.484</td>
<td>236.654</td>
<td>98.426</td>
<td>14.855</td>
<td>25.902</td>
</tr>
<tr>
<td>3</td>
<td>0.484</td>
<td>236.654</td>
<td>98.426</td>
<td>14.855</td>
<td>25.902</td>
</tr>
<tr>
<td>4</td>
<td>0.484</td>
<td>236.654</td>
<td>98.426</td>
<td>14.855</td>
<td>25.902</td>
</tr>
<tr>
<td>5</td>
<td>0.484</td>
<td>236.654</td>
<td>98.426</td>
<td>14.855</td>
<td>25.902</td>
</tr>
<tr>
<td>6</td>
<td>0.484</td>
<td>236.654</td>
<td>98.426</td>
<td>14.855</td>
<td>25.902</td>
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<tr>
<td>7</td>
<td>0.484</td>
<td>236.654</td>
<td>98.426</td>
<td>14.855</td>
<td>25.902</td>
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<tr>
<td>8</td>
<td>0.484</td>
<td>236.654</td>
<td>98.426</td>
<td>14.855</td>
<td>25.902</td>
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<td>9</td>
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<td>236.654</td>
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<td>14.855</td>
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<td>0.484</td>
<td>236.654</td>
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<td>25.902</td>
</tr>
<tr>
<td>11</td>
<td>0.484</td>
<td>236.654</td>
<td>98.426</td>
<td>14.855</td>
<td>25.902</td>
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<tr>
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<td>0.484</td>
<td>236.654</td>
<td>98.426</td>
<td>14.855</td>
<td>25.902</td>
</tr>
</tbody>
</table>

7 Conclusion

The use of auxiliary information on a single auxiliary variable $x$, in the form of a mixed estimator by mixing the ratio estimator with the dual to product estimator for estimating $Y$. In this paper, we extend and refined the idea to utilize the information on two auxiliary variables ($x$ and $z$) in order to estimate $Y$ and proposed a class of estimators $t$ which includes many popular estimators along with classes of estimators as its members notably the classical ratio estimator $t_1$ of [8], class of estimator $t_1$ of [9], $t_2$, $t_3$ and $t_4$ proposed by [10], $t_6$ of [12], $t_8$ of [14] and the class $t_{11}$ of [18].
Table 6. PRE of MVB Estimators of Different Classes

<table>
<thead>
<tr>
<th>P. No.</th>
<th>$t_1$</th>
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<th>$t_{10}$</th>
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<td>1105.691</td>
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<td>235.108</td>
<td>179.206</td>
<td>235.108</td>
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</table>

The MVB estimators of various classes were compared on the basis of their percent relative biases and percent gain in precision with respect to mean per unit estimator $\bar{y}$. Both the empirical and numerical studies advocated in favor of the proposed class of estimators.

**Competing Interests**

Authors have declared that no competing interests exist.

**References**


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